Mechanics

TOPIC 2

How Scientists Study Mechanics

If a moving car is accelerating, is the speed of the car increasing?

Acceleration and velocity are vector quantities, having both magnitude (size) and direction. The acceleration of a car can be produced by a change in the magnitude of the car's velocity (speed) or by a change in the direction of the car's velocity.

Suppose a car is initially traveling at 12.0 meters/second to the east. If the car accelerates at 2.0 meters/second^2 to the east for 4.0 seconds, the final velocity of the car is 20.0 meters/second to the east. The speed of the car increases 8.0 meters/second in this 4.0-second interval. On the other hand, if a car initially traveling at 12.0 meters/second to the east accelerates at 2.0 meters/second^2 to the west for 4.0 seconds, the final velocity of the car is 4.0 meters/second to the east. The speed of the car decreases 8.0 meters/second in this 4.0-second interval even though the car is accelerating.

If a car travels along a horizontal circular path, the car experiences acceleration directed toward the center of the circular path. This acceleration is called centripetal acceleration. Although the car may be traveling at a constant speed, the car's direction of travel continuously changes and thus the car is accelerating.

TOPIC 2

Mechanics

Vocabulary

acceleration
centripetal acceleration
centripetal force
closed system
coefficient of friction
displacement
distance
equilibrium
free fall
free-body diagram
friction
gravitational field
gravitational field strength
gravitational force
gravity
horizontal component
impulse
inertia
instantaneous velocity
kinetic friction
kilogram
law of conservation of momentum
linear motion
mechanics
meter
momentum
net force
newton
normal force
pendulum
period (of a pendulum)
resolution of forces
resultant
second
speed
static friction
tangent
unbalanced force
uniform circular motion
uniform motion
vacuum
vector component
velocity
vertical component
weight

Kinematics

The branch of physics that deals with forces and the way they produce and change motion is called **mechanics.** Kinematics is the mathematical treatment of the motions of bodies without regard to the forces that produce the motion.

Distance and Displacement

When an object moves from one point to another, it experiences a change in position relative to some arbitrary reference point. **Distance** is the total length of a path that an object travels. Distance is a scalar quantity, which means it has magnitude but not direction. **Displacement** is the change in the position of an object described by a vector that begins at the initial position of the object and ends at its final position. Because displacement is a vector quantity, it has both magnitude and direction. Distance and displacement are usually measured in meters, centimeters, or kilometers. The **meter,** m, is the fundamental SI unit of length.

The following example illustrates the difference between distance and displacement. A car is driven on the NYS Thruway from Buffalo to Albany to New York City. The distance traveled by the car is approximately 418 miles or 673 kilometers. The magnitude of the total displacement of the car, however, is only the length of the vector connecting Buffalo and New York City—approximately 313 miles or 504 kilometers. Two or more displacement vectors can be combined to obtain the vector sum, or resultant, as the following sample problem shows.

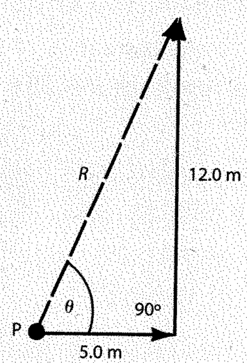
SAMPLE PROBLEM

A student walks 5.0 meters due east and then 12.0 meters due north. Calculate the magnitude and direction of the student's resultant displacement, *R*.

SOLUTION: Identify the known and unknown values.

Known	*Unknown*
$d_1 = 5.0$ m east	$R = ?$ m at ?°
$d_2 = 12.0$ m north	

1. Make a sketch of the situation.



Because east and north are perpendicular to each other, the two displacements form the legs of a right triangle. The resultant displacement is the hypotenuse of the triangle.

2. Write the Pythagorean Theorem.
$c^2 = a^2 + b^2$

3. Solve the equation for *c*.
$c = \sqrt{a^2 + b^2}$

4. Substitute *R* for *c*, then substitute the known values and solve.
$R = \sqrt{(5.0\text{ m})^2 + (12.0\text{ m})^2} = 13\text{ m}$

5. Write a trigonometric function to determine θ.
$\tan\theta = \dfrac{\text{side opposite } \angle\theta}{\text{side adjacent to } \angle\theta}$

6. Substitute the known values and solve.
$\tan\theta = \dfrac{12.0\text{ m}}{5.0\text{ m}}$
$\theta = 67°$
The resultant is 13 meters at 67° north of east.

Displacements along the same straight line can be combined by simple addition or subtraction to find the resultant. If the successive displacements in the previous problem had been 5.0 meters east and 12.0 meters east, the resultant would have been 17.0 meters east. Also, if the student had walked 5.0 meters east, and then 12.0 meters west, the resultant would have been 7.0 meters west.

When successive displacements are not along the same straight line, the resultant can be found either graphically, by making a scaled vector diagram using a metric ruler and a protractor, or algebraically using the law of cosines and the law of sines. Because the laws of cosines and sines are not provided in the *Reference Tables for Physical Setting/Physics*, this type of algebraic solution is not testable.

Speed and Velocity

The position of an object in motion changes with time. The **speed,** v, of an object is the distance that the object moves in a unit of time. Speed is a scalar quantity. The average speed, $\bar{v}$, of an object is given by this formula

$$\bar{v} = \frac{d}{t}$$ Ⓡ

Distance d is in meters and the time interval t is in seconds. The **second,** s, is the fundamental SI unit of time. Thus, the average speed $\bar{v}$ is in meters per second, or m/s, a derived SI unit. If the object's speed is constant

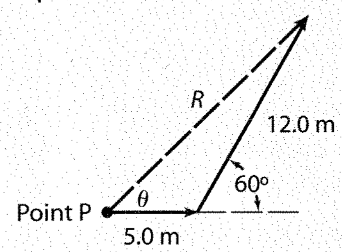
SAMPLE PROBLEM

A person walks 5.0 meters due east and 12.0 meters at 60.° north of east. Find the magnitude and direction of the person's resultant displacement.

SOLUTION: Identify the known and unknown values.

Known	*Unknown*
d_1 = 5.0 m east	R = ? m at ?°
d_2 = 12.0 m at 60.° north of east	

1. Construct a scale drawing. A scale of 1.0 cm = 4.0 m is used, but a scale of 1.0 cm = 2.0 m would provide more accurate results.



Scale: 1.0 cm = 4.0 m

2. Use a ruler to measure the length of the resultant vector.

Resultant vector R measures 3.75 cm.

3. Use the scale of the drawing to convert R in centimeters to meters.

$R = (3.75 \text{ cm})(4.0 \text{ m/cm}) = 15 \text{ m}$

4. Use a protractor to measure θ.

$\theta = 43°$. Thus, the resultant R is 15 m at 43° north of east.

during the entire time interval, $\overline{v}$ is its constant speed, and the object is said to be in **uniform motion.** If the speed of the object varies, the motion is nonuniform.

The **velocity** of an object is the time rate of change of its displacement. Velocity is a vector quantity having direction as well as magnitude. The magnitude of an object's velocity is its speed. For example, if one car travels at 88 kilometers per hour due east and a second car travels at 88 kilometers per hour due north, both cars have the same speed. However, the velocities of the cars differ because the direction of travel is not the same. In physics, the terms speed and velocity are not interchangeable.

Linear motion refers to an object's change of position along a straight line. On a straight path, there are only two possible directions for the velocity. One of these is called the positive direction. The opposite direction, then, is the negative direction. Depending upon the direction of the motion, changes in displacement are also positive or negative. When referring to linear motion in this text, the symbol v is used for both velocity and speed, and the symbol d is used for both displacement and distance.

Graphs of Linear Motion Graphs of position versus time are commonly used to represent the linear motion of an object. The independent variable, time, is recorded on the horizontal axis, and the dependent variable, position, is recorded on the vertical axis. Because $\overline{v} = \frac{d}{t}$, the magnitude of the slope of a position versus time graph at any point equals the object's speed at that instant, and the algebraic sign of the slope indicates whether the velocity is in the positive or negative direction. A straight line indicates constant velocity. A straight horizontal line represents zero velocity, that is, an object at rest. If a position-time graph is a curved line, the velocity is not constant. The slope of the tangent to the curve at any point is called the instantaneous velocity of the object. The **tangent** to a curve at any point on the curve is defined as the line passing through the point and having a slope equal to the slope of the curve at that point. **Instantaneous velocity** is the velocity of an object at any particular instant in time. The term is applied to the motion of an object that is not traveling at constant velocity. The steeper the slope of a position versus time graph, the greater the instantaneous speed. Figure 2-1 shows examples of graphs of linear motion.

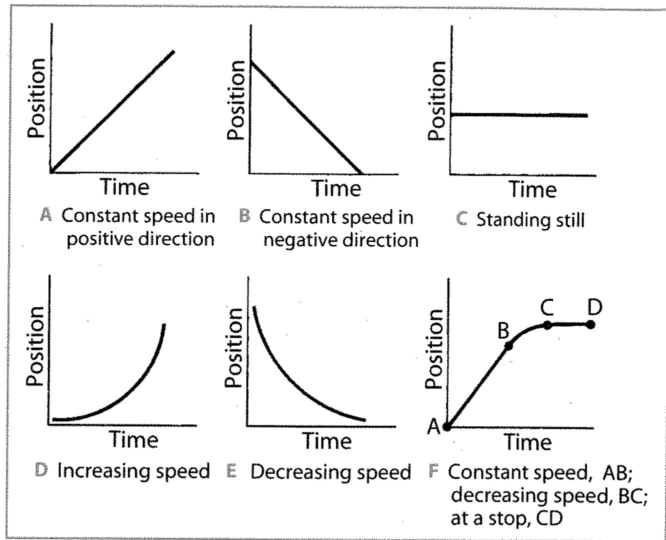


Figure 2-1. Graphs of linear motion, drawn on position-time axes

Acceleration

The time rate of change of velocity is **acceleration,** a, a vector quantity represented by this formula.

$$a = \frac{\Delta v}{t}$$ Ⓡ

The change in velocity Δv is in meters per second and t is the time interval in seconds. Thus, acceleration can be expressed with the unit meters per second per second, or meters per second^2, m/s^2.

Note that the formula, as written without a bar over the a to indicate average, implies constant, or uniform, acceleration. This text does not address nonuniform acceleration.

The average speed $\bar{v}$ of an object accelerating uniformly from an initial speed v_i to a final speed v_f is given by this formula.

$$\bar{v} = \frac{v_i + v_f}{2}$$

This formula is valid only when the acceleration is constant. This formula does not appear on the *Reference Tables for Physical Setting/Physics*, but it is acceptable to use where appropriate.

Velocity versus time graphs can be used to represent accelerated linear motion, as shown in Figure 2-2. The independent variable, time, is measured on the horizontal axis, and the dependent variable, velocity, is recorded on the vertical axis. Because $a = \frac{\Delta v}{t}$, the magnitude of the slope of a velocity versus time graph at any point equals the object's acceleration at that instant, and the algebraic sign of the slope indicates whether the acceleration is in the positive or negative direction. For example, a horizontal line with zero slope indicates constant speed or no acceleration. A straight line with a positive slope shows increasing speed or constant acceleration. A straight line with negative slope shows decreasing speed or constant negative acceleration (deceleration).

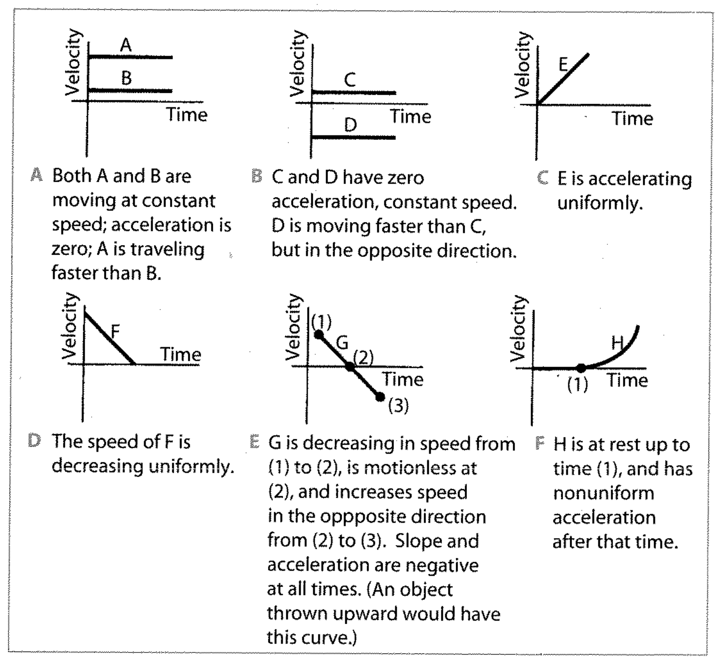


Figure 2-2. Graphs of various types of motion in a straight line path, drawn on velocity-time axes

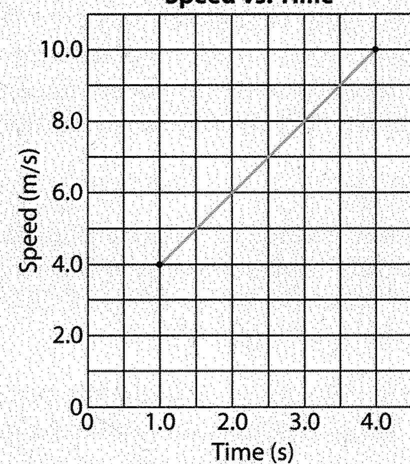
A line that intersects the horizontal time axis indicates a change in direction, that is, the speed in one direction decreases to zero at the time when the graph line intersects the horizontal axis, and then the speed increases in the opposite direction. A curved velocity-time line indicates that acceleration is not constant.

SAMPLE PROBLEM

The graph represents the relationship between the speed of a child coasting downhill on a skateboard and elapsed time. At 1.0 second the child is traveling at 4.0 meters per second, and at 4.0 seconds her speed is 10.0 meters per second.

(a) Calculate the magnitude of the child's acceleration.
(b) Determine the average speed of the child.
(c) Calculate the total distance traveled by the child during this 3.0-second interval.

Speed vs. Time



SOLUTION: Identify the known and unknown values.

Known	*Unknown*
$t_i = 1.0$ s	$a = ?$ m/s^2
$t_f = 4.0$ s	$\bar{v} = ?$ m/s
$v_i = 4.0$ m/s	$d = ?$ m
$v_f = 10.0$ m/s	

1. The slope of the graph is the magnitude of the child's acceleration. Determine the slope by dividing the change in speed by the change in time.

$$a = \frac{\Delta v}{t} = \frac{10.0 \text{ m/s} - 4.0 \text{ m/s}}{4.0 \text{ s} - 1.0 \text{ s}} = 2.0 \text{ m/s}^2$$

2. Write the formula for average speed.

$$\bar{v} = \frac{v_i + v_f}{2}$$

Substitute the known values and solve.

$$\bar{v} = \frac{4.0 \text{ m/s} + 10.0 \text{ m/s}}{2} = 7.00 \text{ m/s}$$

The average speed is the vertical coordinate of the midpoint of the graphed line segment.

3. Write the formula that relates distance, average speed, and time.

$$\bar{v} = \frac{d}{t}$$

Solve the equation for d.

$$d = \bar{v}t$$

Substitute the known values and solve.

$$d = (7.00 \text{ m/s})(3.0 \text{ s}) = 21 \text{ m}$$

In the sample problem above, the distance traveled by the child could have been found by determining the area under the graph line. That area has the shape of a trapezoid, which can be separated into a rectangle and a triangle, as shown in Figure 2-3.

Recall that the formula for the area of a rectangle is $\boldsymbol{A = bh}$ and that the formula for the area of a triangle is $\boldsymbol{A = \frac{1}{2}bh}$ where b is the base and h is the height. Thus, the area under the line is the sum of the area of the rectangle and the area of the triangle. For the purpose of this calculation, these quantities may be represented as meters.

That is,

$$A_{\text{rectangle}} = bh = (3.0\text{ s})(4.0\text{ m/s}) = 12\text{ m}$$

$$A_{\text{triangle}} = \frac{1}{2}bh = \frac{1}{2}(3.0\text{s})(6.0\text{ m/s}) = 9.0\text{ m}$$

$$A_{\text{total}} = 12\text{ m} + 9.0\text{ m} = 21\text{ m}$$

The total distance is 21 meters, as calculated in the sample problem. Thus, the physical significance of the area under the line of a speed versus time graph is the distance traveled.

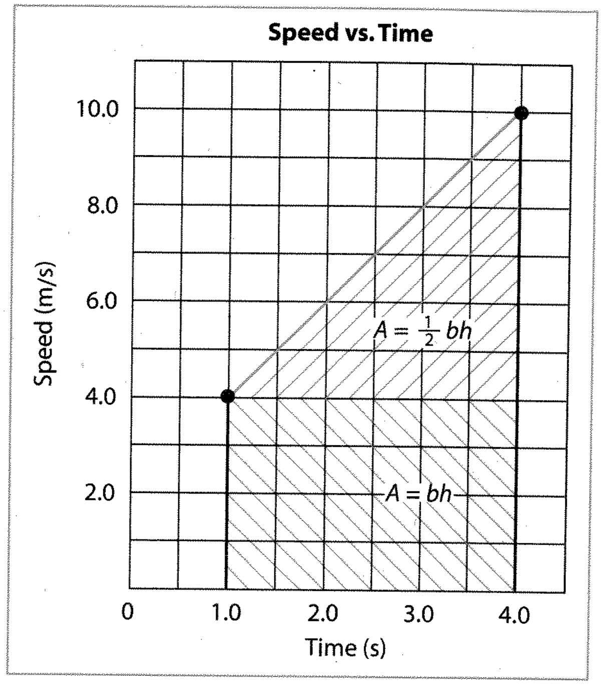


Figure 2-3. The area under a speed vs. time graph: The magnitude of the area of the rectangle is 12 m and the magnitude of the area of the triangle is 9 m. The magnitude of the total area is 21 m, which is the total distance in meters traveled by the child.

Final Velocity and Distance Traveled During Constant Acceleration

Acceleration is defined by the formula $a = \frac{\Delta v}{t}$. Because Δ always represents a change in a variable, that is, final conditions minus initial conditions, it follows that $a = \frac{v_f - v_i}{t}$. Solving for the final speed v_f yields this formula.

$$v_f = v_i + at$$

This expression can be combined with $d = \overline{v}t$ and $\overline{v} = \frac{v_i + v_f}{2}$ to obtain a useful expression for displacement d that involves initial velocity v_i, the acceleration a, and time t.

$$d = \overline{v}t = \left(\frac{v_i + v_f}{2}\right)t = \frac{1}{2}(v_i + v_f)t = \frac{1}{2}(v_i + v_i + at)t$$

Thus, the equation becomes the following.

$$d = v_it + \frac{1}{2}at^2$$

This formula is valid only when acceleration is constant.

The velocity of an object as a function of its displacement can be determined without knowing the elapsed time. From the previous derivation, the following equation can be written.

$$d = \frac{1}{2}(v_i + v_f)t$$

Solving the equation $v_f = v_i + at$ for t yields

$$t = \frac{v_f - v_i}{a}$$

Combining these expressions yields

$$d = \frac{1}{2}(v_f + v_i)\left(\frac{v_f - v_i}{a}\right) = \frac{v_f^2 - v_i^2}{2a}$$

Solving for the final velocity yields

$$v_f^2 = v_i^2 + 2ad$$

This formula is valid only for constant acceleration.

In many problems involving motion, the object is initially at rest and v_i is zero. In such cases, terms containing v_i drop out of the motion formulas

SAMPLE PROBLEM

A car is originally traveling at 15.0 meters per second (approximately 34 miles per hour) on a straight, horizontal road. The driver applies the brakes, causing the car to decelerate uniformly at 4.00 meters per second^2 until it comes to rest. Calculate the car's stopping distance.

SOLUTION: Identify the known and unknown values.

Known	*Unknown*
$v_i = 15.0$ m/s	$d = ?$ m
$v_f = 0.0$ m/s	
$a = -4.00$ m/s^2	

The acceleration is negative because its direction is opposite the direction of the moving car.

1. Write the formula that relates initial and final velocities, acceleration, and distance.

$$v_f^2 = v_i^2 + 2ad$$

2. Solve the equation for distance, d.

$$d = \frac{v_f^2 - v_i^2}{2a}$$

3. Substitute the known values and solve.

$$d = \frac{(0.0\text{ m/s})^2 - (15.0\text{ m/s})^2}{2(-4.00\text{ m/s}^2)}$$

$$d = \frac{-225\text{ m}^2/\text{s}^2}{-8.00\text{ m/s}^2} = 28.1\text{ m}$$

ALTERNATE SOLUTION:

1. Write the formula that defines acceleration

$$a = \frac{\Delta v}{t}.$$

2. Solve the equation for t.

$$t = \frac{\Delta v}{a}$$

3. Substitute the known values and solve.

$$t = \frac{0.0\text{ m/s} - 15.0\text{ m/s}}{-4.00\text{ m/s}^2} = 3.75\text{ s}$$

4. Write the formula that relates distance, initial velocity, acceleration, and time.

$$d = v_i t + \frac{1}{2}at^2$$

5. Substitute the known values and solve for d.

$$d = (15.0\text{ m/s})(3.75\text{ s}) + \tfrac{1}{2}(-4.00\text{ m/s}^2)(3.75\text{ s})^2$$

$$d = 56.3\text{ m} + (-28.1\text{ m}) = 28.2\text{ m}$$

If the initial speed was doubled to 30.0 meters per second (or 67 miles per hour), the stopping distance would quadruple!

and the equations are simplified. In addition, the symbol for v_f can be written as v. Thus, for objects starting from rest and accelerating uniformly,

$$a = \frac{v}{t} \qquad d = \frac{1}{2}at^2 \qquad \bar{v} = \frac{v}{2} \qquad v^2 = 2ad$$

Freely Falling Objects

In a **vacuum,** which is a space in which there is no matter, a coin and a feather fall with the same acceleration due to gravity, g. **Gravity** is the force between the mass of Earth and the mass of any object in the vicinity of Earth. According to the *Reference Tables for Physical Setting/Physics,* near the surface of Earth g is a constant 9.81 meters per second^2. The ideal falling motion of an object acted upon only by the force of gravity is called **free fall.** Ⓡ

Although the acceleration due to gravity is the same for all objects in a vacuum, in air the acceleration of the feather is less than that of the coin because the shape and exposed area of the feather result in greater air resistance.

If an object falls freely from rest (air resistance is neglected), its speed and position at any instant in time are given by $v = gt$ and $d = \frac{1}{2}gt^2$. Table 2-1 shows the speed and distance traveled by an object falling freely from rest near Earth's surface in the absence of air resistance.

Table 2-1. Free Fall of an Object Starting from Rest

Time of fall (s)	Speed (m/s)	Distance traveled (m)
0.00	0.00	0.00
1.00	9.81	4.91
2.00	19.6	19.6
3.00	29.4	44.1
4.00	39.2	78.5
5.00	49.1	123

Review Questions

1. If a boy runs 125 meters north, and then 75 meters south, his total displacement is

(1) 50. m north (2) 50. m south (3) 200. m north (4) 200. m south

2. A student walks 3 blocks south, 4 blocks west, and 3 blocks north. What is the resultant displacement of the student?

(1) 10. blocks east (2) 10. blocks west (3) 4 blocks east (4) 4 blocks west

3. A girl attempts to swim directly across a stream 15 meters wide. When she reaches the other side, she is 15 meters downstream. Calculate the magnitude of her displacement.

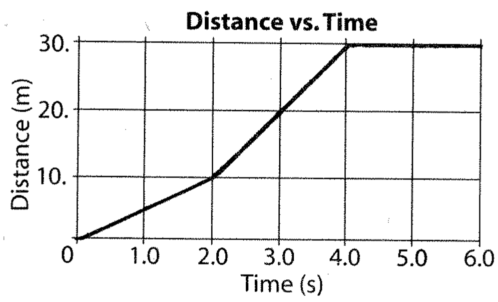
4. What is the average speed of an object that travels 6.00 meters north in 2.00 seconds and then travels 3.00 meters east in 1.00 second?

(1) 9.00 m/s (2) 0.333 m/s (3) 3.00 m/s (4) 4.24 m/s

5. In a 4.0-kilometer race, a runner completes the first kilometer in 5.9 minutes, the second kilometer in 6.2 minutes, the third kilometer in 6.3 minutes, and the final kilometer in 6.0 minutes. The average speed of the runner for the race is approximately

(1) 0.16 km/min (2) 0.33 km/min (3) 12 km/min (4) 24 km/min

6. The graph below shows the relationship between the position of an object moving in a straight line and elapsed time.



What is the speed of the object during the time interval $t = 2.0$ seconds to $t = 4.0$ seconds?

(1) 0.0 m/s (2) 5.0 m/s (3) 7.5 m/s (4) 10. m/s

7. A particle is accelerated uniformly from rest to a speed of 50. meters per second in 5.0 seconds. The average speed of the particle during this 5.0-second time interval is

(1) 5.0 m/s (2) 10. m/s (3) 25 m/s (4) 50. m/s

8. Which statement best describes the movement of an object with zero acceleration?

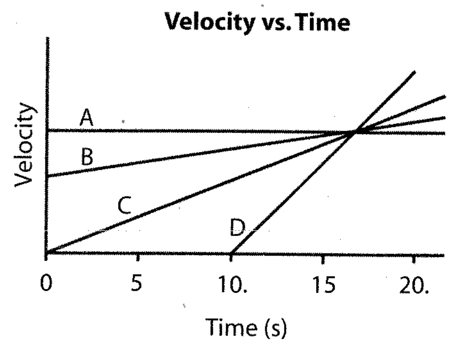
(1) The object must be at rest.
(2) The object must be slowing down.
(3) The object may be speeding up.
(4) The object may be in motion.

9. A particle has a constant acceleration of 2.0 meters per second^2. Calculate the time required for the particle to accelerate from 8.0 meters per second to 28 meters per second.

10. If an object is traveling east with a decreasing speed, the direction of the object's acceleration is

(1) north (2) south (3) east (4) west

Base your answers to questions 11 and 12 on the following graph, which represents the relationship between velocity and time of travel for four cars, A, B, C, and D, in straight-line motion.



11. Which car has the greatest acceleration during the time interval 10. seconds to 15 seconds?

12. Which car travels the greatest distance during the time interval 0 second to 10. seconds?

(1) A only
(2) B only
(3) C only
(4) The distance traveled is the same for cars A, B, and C.

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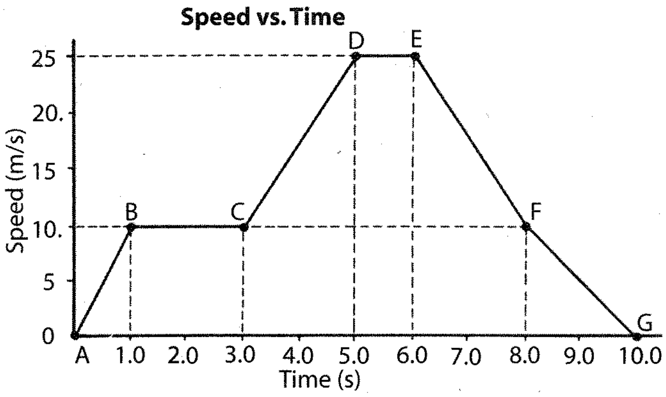
13. Starting from rest, an object rolls freely down a 10.-meter long incline in 2.0 seconds. The acceleration of the object is

(1) 5.0 m/s (2) 5.0 m/s^2 (3) 10. m/s (4) 10. m/s^2

14. A car accelerates uniformly from rest at 3.2 meters per second^2. Calculate the speed of the car when it has traveled a distance of 40. meters.

Base your answers to questions 15 through 19 on the graph below, which represents the relationship between speed and time for an object in straight-line motion.



15. Calculate the acceleration of the object during the time interval $t = 3.0$ seconds to $t = 5.0$ seconds.

16. Determine the average speed of the object during the time interval $t = 6.0$ seconds to $t = 8.0$ seconds.

17. Calculate the total distance traveled by the object during the first 3.0 seconds.

18. Identify the interval during which the magnitude of the object's acceleration is greatest.

19. During the interval $t = 8.0$ seconds to $t = 10.0$ seconds, the speed of the object is

(1) zero
(2) increasing
(3) decreasing
(4) constant, but not zero

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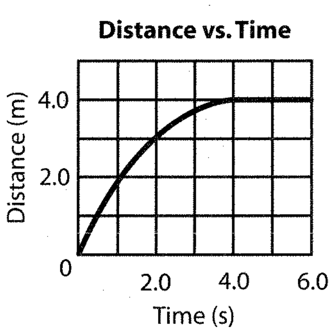
20. The graph below represents the relationship between distance and time of travel for an object moving in a straight line.

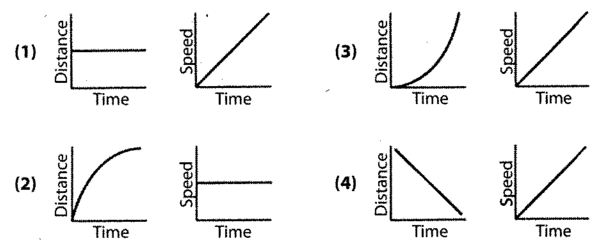


Determine the instantaneous speed of the object at 1.5 seconds.

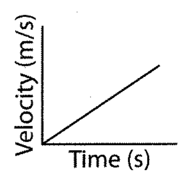
21. A boat heads directly eastward across a river at 12 meters per second. If the current in the river is flowing at 5.0 meters per second due south, what is the magnitude of the boat's resultant velocity?

(1) 7.0 m/s (2) 8.5 m/s (3) 13 m/s (4) 17 m/s

22. Which pair of graphs represents the same motion?



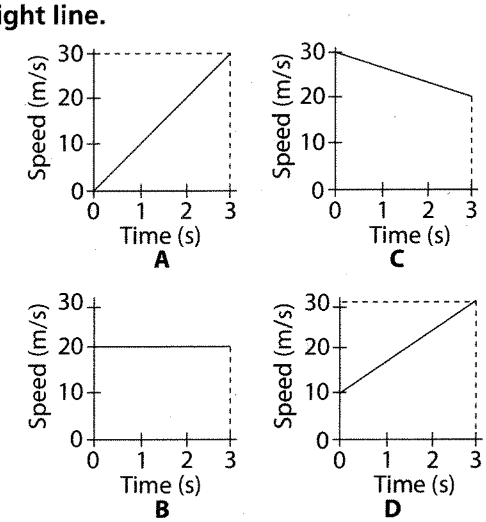
23. The graph below represents the motion of a body moving along a straight line.



Which quantity related to the motion of the body is constant?

(1) speed
(2) velocity
(3) acceleration
(4) displacement

Base your answers to questions 24 through 28 on the following four graphs, which represent the relationship between speed and time for four different objects, A, B, C, and D moving, in a straight line.



24. Which object had a retarding force acting on it?

25. Which object was *not* accelerating?

26. Which object traveled the greatest distance in the 3.0-second time interval?

27. Which object had the greatest acceleration?

28. Compared to the average speed of object A, the average speed of object D is

(1) less
(2) greater
(3) the same

29. An object initially traveling at 20. meters per second west accelerates uniformly at 4.0 meters per second^2 east for 2.0 seconds. The displacement of the object during these 2.0 seconds is

(1) 32 m east
(2) 32 m west
(3) 48 m east
(4) 48 m west

30. An object initially traveling at 20. meters per second south accelerates uniformly at 6.0 meters per second^2 north and is displaced 25 meters. The final velocity of the object is

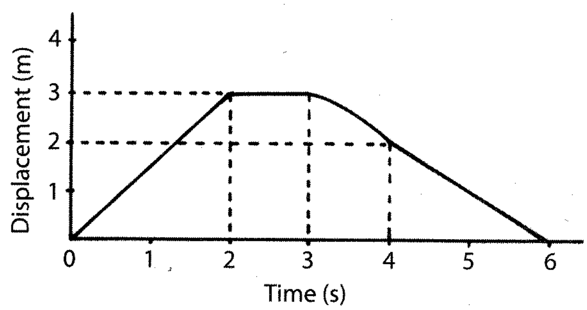
(1) 26 m/s north
(2) 26 m/s south
(3) 10. m/s north
(4) 10. m/s south

31. The time-rate of change of displacement is

(1) acceleration
(2) distance
(3) speed
(4) velocity

Base your answers to questions 32 through 35 on the following graph, which represents the relationship between the displacement of an object and time.

Displacement vs. Time



32. How far is the object from the starting point at the end of 3 seconds?

33. During which time interval is the object at rest?

34. What is the average velocity of the object from $t = 0$ to $t = 3$ seconds?

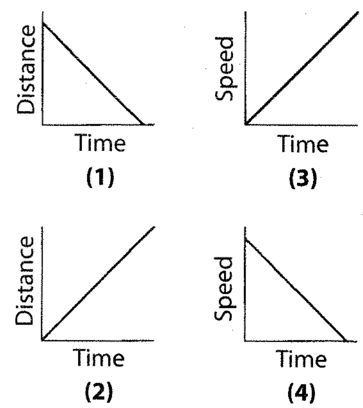
(1) 1 m/s (2) 2 m/s (3) 3 m/s (4) 0 m/s

35. During which time interval is the object accelerating?

36. Which quantity is constant for a freely falling object near Earth's surface?

(1) displacement
(2) speed
(3) velocity
(4) acceleration

37. Which graph best represents the motion of an object falling from rest near Earth's surface? [Neglect friction.]

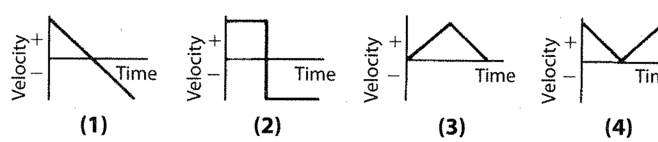


38. What is the total distance that an object near the surface of Earth falling freely from rest travels in 3.0 seconds?

(1) 88 m (2) 44 m (3) 29 m (4) 9.8 m

39. An object starts from rest and falls freely near Earth's surface for 3.00 seconds. Calculate the final speed of the object.

40. An object is thrown vertically upward from the surface of Earth. Which graph best represents the relationship between velocity and time for the object as it rises and then returns to Earth?



Statics

The branch of mechanics that treats forces which act on objects at rest is called <u>statics</u>. A force is a push or pull measured in newtons, N, a derived unit in the SI system. Force is a vector quantity.

Concurrent Forces

Two or more forces that act on the same object at the same time are called concurrent forces. The single force that is equivalent to the combined effect

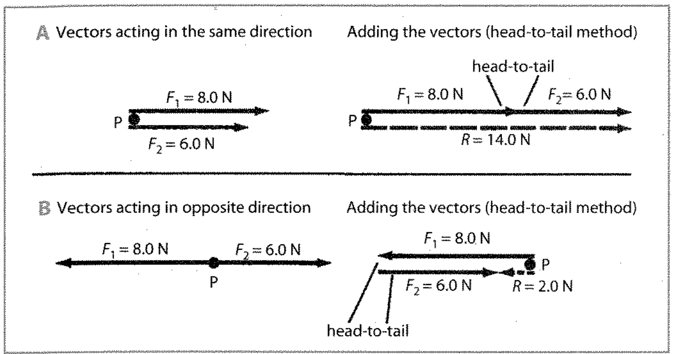


Figure 2-4. The resultants *R* of concurrent forces acting along the same straight line: (A) acting in the same direction, and (B) acting in opposite directions

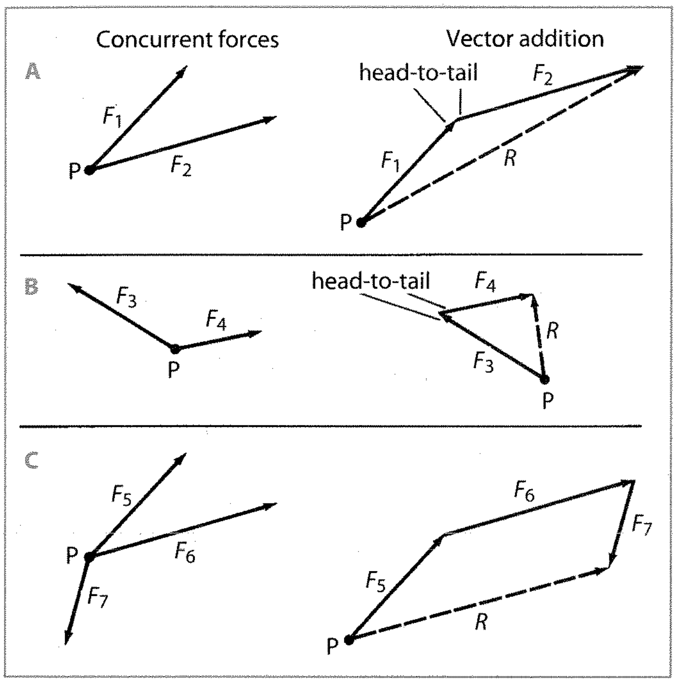


Figure 2-5. Adding force vectors by the triangle method

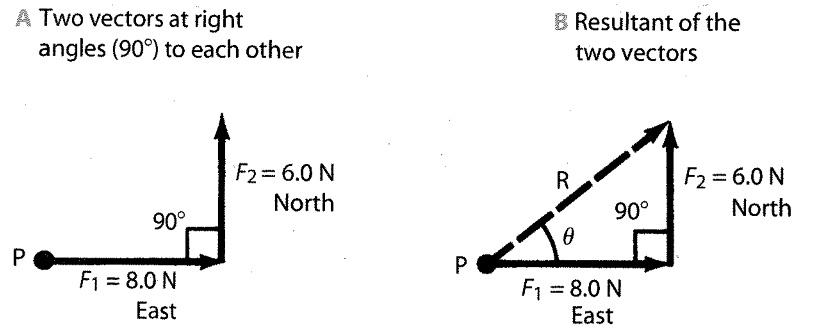
of these concurrent forces is called the **resultant.** The process of combining the magnitude and direction of concurrent forces to determine their resultant is called the composition of forces. If two concurrent forces F_1 and F_2 act in the same direction, the angle between the forces is 0° and the resultant force is the sum, $F_1 + F_2$, of their magnitudes acting in the same direction as the individual forces. This is the largest resultant the two forces can have. If the two forces act in opposite directions, the angle between the forces is 180.° and the resultant force is the difference, $F_1 - F_2$, between their magnitudes, acting in the direction of the larger force. This is the smallest resultant the two forces can have. Figure 2-4 illustrates these concepts for an 8.0-newton force and a 6.0-newton force acting concurrently on point P.

Triangle Method of Adding Concurrent Forces The resultant of two concurrent forces F_1 and F_2 acting at an angle between 0° and 180.° can be found by the triangle method of vector addition. In this method, each force is represented by a vector drawn to scale, with its length corresponding to the magnitude of the force and its direction corresponding to the direction of the force. To add the two vectors, place the tail of the second vector F_2 at the head of the first vector F_1. The resultant is the vector drawn from the tail of F_1 to the head of F_2, as shown in Figure 2-5.

The magnitude of the resultant is found by measuring the vector length with a ruler and then multiplying by the scale. The direction of the vector is determined by using a protractor to measure the angle of the resultant with respect to a compass point. In Figure 2-5A, the resultant has a greater magnitude than either force. In Figure 2-5B, the magnitude of the resultant is smaller than either force. Figure 2-5C correctly implies that the resultant of any number of concurrent forces acting on an object can be found by adding their vectors head to tail. The final resultant is the net force acting on the object. The net force is the single force that is equivalent to the combined effect of concurrent forces acting on an object.

If two concurrent forces act at right angles to each other, the head to tail method of vector addition produces a right triangle in which the hypotenuse is the resultant. In this case, the resultant vector is often found algebraically using the Pythagorean theorem, $c^2 = a^2 + b^2$. Figure 2-6 illustrates this method for an 8.0-newton force to the east and a 6.0-newton force to the north acting concurrently on point P.

Parallelogram Method of Adding Concurrent Forces An alternate graphical method for determining the vector sum of two concurrent forces acting



Resultant from the Pythagorean theorem

$R^2 = F_1^2 + F_2^2$

$R = \sqrt{64\ N^2 + 36\ N^2} = 10.\ N$

The tangent of the angle θ is $\frac{6.0}{8.0}$, or 0.75, making the angle equal to 37°.

Figure 2-6. Finding the resultant of two concurrent forces acting at right angles (90°) to each other

at any angle is the parallelogram method shown in Figure 2-7. The two vectors are drawn to scale with both tails originating at the same point. A parallelogram is then constructed with the force vectors as adjacent sides. Recall that a parallelogram is a quadrilateral having opposite sides parallel and equal in length. The diagonal of the parallelogram drawn from the vertex of the original two vector tails is the resultant.

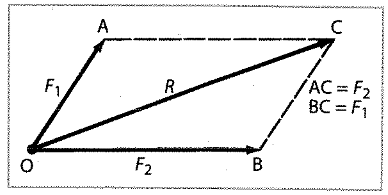


Figure 2-7. Finding the resultant of two concurrent forces at any angle to each other using the parallelogram method

The parallelogram method makes it obvious that as the angle between two vectors increases from 0° to 180.° the magnitude of the resultant decreases from a maximum, $F_1 + F_2$ at 0°, to a minimum, $F_1 - F_2$ at 180°.

Resolution of Forces

Just as force vectors can be added to provide the magnitude and direction of the resultant force, force vectors can be resolved or broken up into component vectors. The process of determining the magnitude and direction of the components of a force is called **resolution of forces.** Although a force vector could be resolved into any number of components, it is usually resolved into two components that are perpendicular to each other. The **vector components** of a force vector F are the concurrent forces whose vector sum is F. If the force is resolved into two components at right angles to each other, vector F is a diagonal of the rectangle formed in the parallelogram method. Perpendicular component forces are usually given directions such as east-west and north-south, perpendicular and parallel to the ground, or perpendicular and parallel to an incline.

Graphical Method of Resolving a Force into Components Figure 2-8 shows a 50.-newton force at 37° north of east being resolved into two perpendicular component forces by the graphical method. Force F_1 is along the north-south axis and force F_2 is along the east-west axis. The magnitude of each force is found by drawing a perpendicular to each axis from the head end of the given vector. The line drawn from the origin, O, to each intersection with the axes determines the magnitude of each component vector. The components are measured to be $F_1 = 30.$ N north and $F_2 = 40.$ N east. Note that the vector sum of the components is equal to the original force F.

Algebraic Method of Resolving a Force into Components It is also possible to determine algebraically the perpendicular components of a force or any other vector. Figure 2-9 shows how vector A, which is at an angle θ with the horizontal, can be resolved into components at right angles to each other. Recall that in a right triangle, the sine of one of the acute angles is the ratio of the side opposite the angle to the hypotenuse and that the

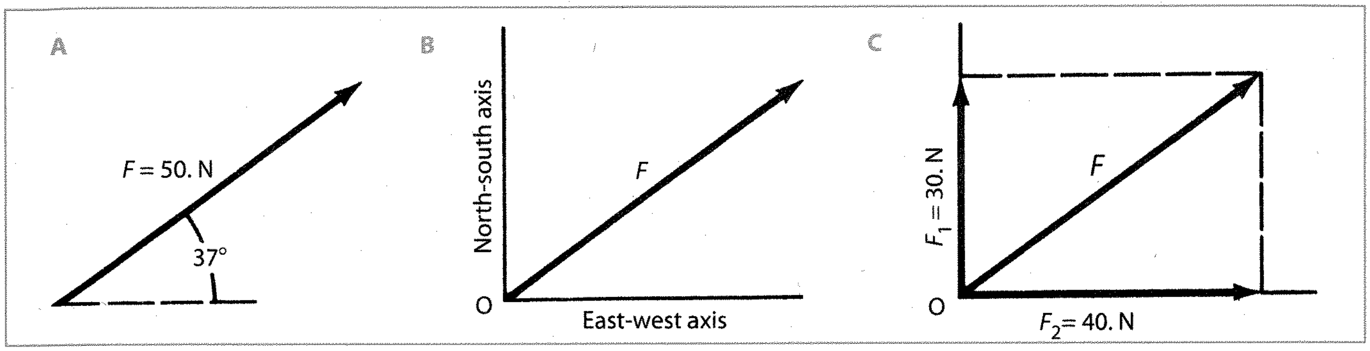


Figure 2-8. Resolution of a force into two components at right angles (90°) to each other: (A) **The vector to be resolved is a force vector F of 50. N directed at 37° north of east.** (B) **Horizontal (east-west) and vertical (north-south) axes are constructed at the tail of the vector.** (C) **Dashed lines that start at the head of vector F and extend perpendicularly to the axes define two new vectors F_1 and F_2 that are the vertical and horizontal components of the original force vector F. To the scale of the drawing, F_1 measures 30. N north and F_2 measures 40. N east.**

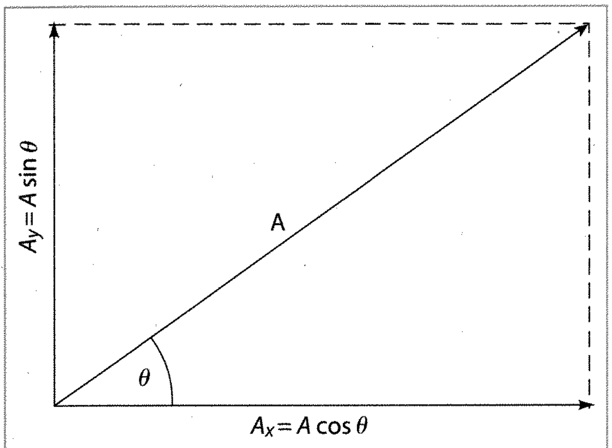


Figure 2-9. A force vector *A* resolved into horizontal and vertical components

cosine of the angle is the ratio of the adjacent side of the angle to the hypotenuse. Thus, for any vector A, making angle θ with the horizontal, the following apply.

$$A_x = A \cos \theta$$
$$A_y = A \sin \theta$$

Thus, the components of a vector can be readily determined without making a scale drawing.

Component forces have practical applications, such as pushing a lawnmower or pulling a suitcase by an extended handle at constant speed along the ground. In pulling a suitcase, the magnitude of the force that needs to be exerted depends upon the angle of the extended handle with the ground. The suitcase is moved only by the component of the applied force parallel to the ground. This component is a smaller fraction of the applied force when the angle the extended handle makes with the ground is larger. Thus, a greater force must be applied as the angle between the extended handle and the ground becomes larger.

Equilibrium

The vector sum of the concurrent forces acting on an object is called the **net force,** F_{net}. If the net force acting on an object is zero, the object is in **equilibrium.** An object at rest is said to be in static equilibrium.

In the example illustrated in Figure 2-6, the resultant of an 8.0-newton force to the east and a 6.0-newton force to the north is a 10.-newton force at 37° north of east. If a third force of 10. newtons acting at 37° south of west was applied, the net force would be zero. A force that is equal in magnitude and opposite in direction to the resultant of concurrent forces produces equilibrium.

Figure 2-10A shows a sign hanging from the side of a building. Because the sign is at rest, it is in static equilibrium and the net force on the sign is zero. But three forces are acting on the sign. They are its weight F_g acting perpendicular to the ground, the force exerted by the cable F_1 pulling in the direction of the cable toward the building, and the outward push F_2 of the horizontal rod.

Figure 2-10B is a free-body diagram for the sign. A **free-body diagram** is a sketch, or scale drawing, that shows all the forces acting concurrently on an object. In this example, the weight of the sign must be equal in magnitude and opposite in direction to the vector sum or resultant of the forces exerted by the rod and the cable. This diagram, drawn to scale, indicates that the weight of the sign is smaller than the magnitude of either of the forces used to support it from the building.

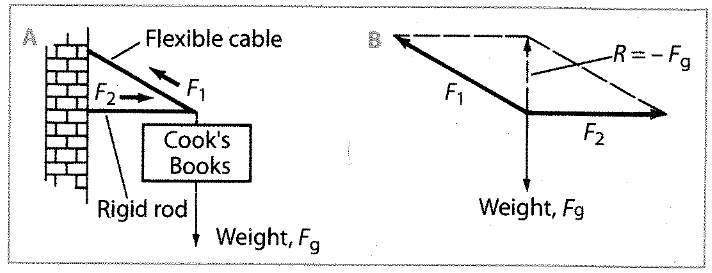
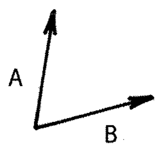


Figure 2-10. (A) **The sign is supported by a flexible cable and a rigid rod.** (B) **A free-body diagram showing the relationships among the forces on the sign while it is in static equilibrium**

Review Questions

41. The vector diagram below shows two concurrent forces, *A* and *B*.



Which vector diagram best represents the resultant force?

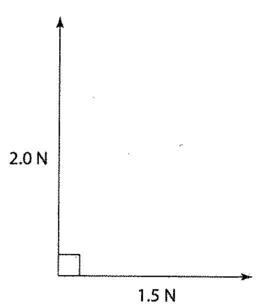
(1) 

(3) 

(2) 

(4) 

42. The vector diagram below represents two concurrent forces.



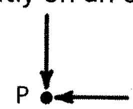
What is the magnitude of the resultant force?

(1) 2.5 N (2) 3.5 N (3) 3.0 N (4) 4.0 N

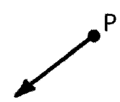
43. The magnitude of the resultant of two concurrent forces is a minimum when the angle between them is

(1) 0° (2) 45° (3) 90.° (4) 180.°

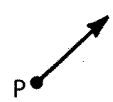
44. The vector diagram below represents two forces acting concurrently on an object at point *P*.

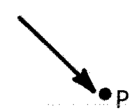


Which vector diagram best represents the resultant force?

(1) 

(3) 

(2) 

(4) 

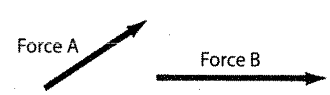
45. As the angle between two concurrent forces of 5.0 newtons and 7.0 newtons increases from 0° to 180.°, the magnitude of their resultant changes from

(1) 0.0 N to 35 N (3) 12.0 N to 2.0 N
(2) 2.0 N to 12.0 N (4) 12.0 N to 0.0 N

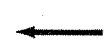
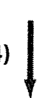
46. A resultant force with a magnitude of 20. newtons can be produced by two concurrent forces having magnitudes of

(1) 5.0 N and 10. N (3) 20. N and 50. N
(2) 20. N and 20. N (4) 30. N and 5.0 N

47. The diagram below represents two force vectors, *A* and *B*.



Which vector best represents the force that could act concurrently with force A to produce force B?

(1) 
(2) 
(3) 
(4) 

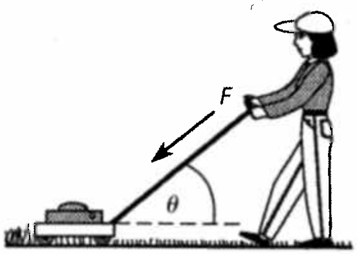
48. Three forces with magnitudes of 10. newtons, 8 newtons, and 6 newtons acting concurrently on an object produce equilibrium. The resultant of the 6-newton and 8-newton forces has a magnitude of

(1) 0.0 N
(2) between 0.0 N and 10. N
(3) 10. N
(4) more than 10. N

49. What is the total number of components into which a single force can be resolved?

(1) an unlimited number
(2) two components
(3) three components
(4) four components at right angles to each other

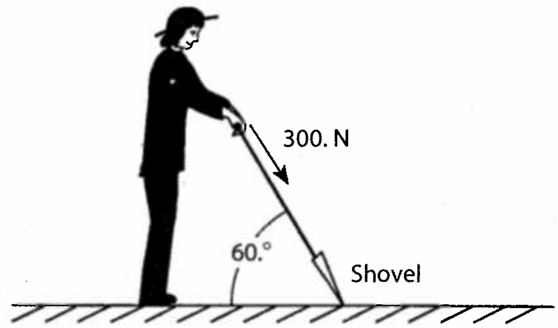
50. A lawnmower is pushed with a constant force *F*, as shown in the following diagram.



As angle θ between the lawnmower handle and the horizontal increases, what happens to the horizontal and vertical components of *F*?

(1) The horizontal component decreases and the vertical component decreases.
(2) The horizontal component decreases and the vertical component increases.
(3) The horizontal component increases and the vertical component decreases.
(4) The horizontal component increases and the vertical component increases.

51. The following diagram shows a person exerting a 300.-newton force on the handle of a shovel that makes an angle of 60.° with the horizontal ground.



Calculate the magnitude of the component of the force perpendicular to the ground.

52. A vector makes an angle θ with the horizontal. The horizontal and vertical components of the vector will be equal in magnitude if angle θ is

(1) 30.° (2) 45° (3) 60.° (4) 90.°

53. Which terms represent a vector quantity and the scalar quantity of the vector's magnitude, respectively?

(1) acceleration and velocity
(2) mass and force
(3) speed and time
(4) displacement and distance

54. The fundamental units for a force of one newton are

(1) meters / second^2
(2) kilograms
(3) meters / second^2 / kilogram
(4) kilogram • meters / second^2

Dynamics

The branch of mechanics that deals with how the forces acting on an object affect its motion is called <u>dynamics</u>. The physical laws that govern dynamics were formulated by Isaac Newton.

Newton's Three Laws of Motion

Recall that when the net force acting on an object is zero, it is said to be in equilibrium. An object in static equilibrium is at rest. An object in dynamic equilibrium moves with a constant velocity, that is, at constant speed in a straight line.

Newton's First Law According to Newton's first law, an object maintains a state of equilibrium, remaining at rest or moving with constant velocity, unless acted upon by an unbalanced force. An **unbalanced force** is a nonzero net force acting on an object. According to the first law, an unbalanced force always produces a change in an object's velocity, a vector quantity. This change in velocity produces an acceleration because the object's speed, or direction of motion, or both speed and direction are changing. The inertia of an object is independent of its speed or velocity.

The law of inertia is another name for the first law. **Inertia** is the resistance of an object to a change in its motion. The inertia of an object is directly proportional to its mass. The fundamental SI unit of mass is the **kilogram,** kg. Mass is a scalar quantity.

Inertia can have a devastating effect on a person not wearing a seat belt in a car traveling at high speed. If the car runs off the road and collides with a tree, the force of the collision causes the car to rapidly decelerate. However, the force does not act on the passengers in the car. They continue to move with the same velocity as before the collision, until they are decelerated by colliding with the dashboard or front window. When seat belts are used, the passengers are fastened to the car and decelerate upon impact at the same rate as the car.

Newton's Second Law According to Newton's second law, when an unbalanced force acts on an object, the object is accelerated in the same direction as the force. The acceleration is directly proportional to the magnitude of the unbalanced force and inversely proportional to the mass of the object, as shown in this formula.

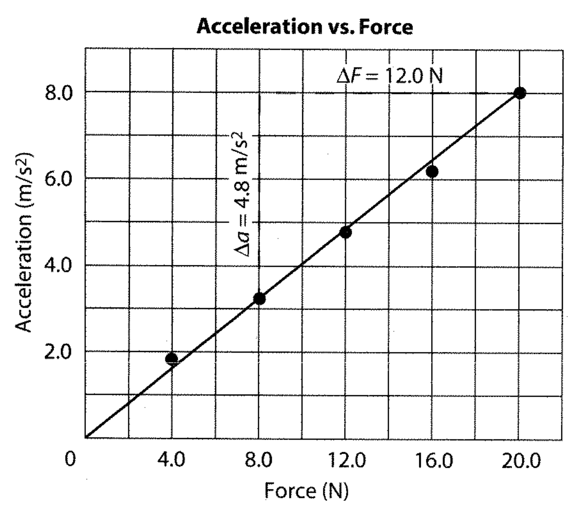
$$a = \frac{F_{net}}{m}$$ (R)

Mass m is in kilograms, acceleration a is in meters per second^2, and the net force F_{net} is in newtons. One **newton** is equal to the force that imparts an acceleration of one meter per second^2 to a one-kilogram mass. The newton, N, is the derived SI unit of force. One newton equals one kilogram • meter per second^2, kg • m/s^2.

Simple laboratory experiments can be performed to verify Newton's second law. In one experiment, the net force on a cart originally at rest on a horizontal surface is varied and the resulting acceleration calculated by timing the motion of the cart for some distance. The data is plotted with force as the independent variable on the horizontal axis and acceleration as the dependent variable on the vertical axis. Figure 2-11 shows such a graph.

The slope of the line of best fit is $\frac{\Delta a}{\Delta F}$ and is equal to $\frac{1}{m}$. Therefore, the reciprocal of the slope of the line of best fit is the mass of the object being accelerated. Sometimes the axes are reversed so that the mass of the object can be determined directly from the slope.

Newton's Third Law According to Newton's third law, when one object exerts a force on a second object, the second object exerts a force on the first that is equal in magnitude and



$$\text{slope} = \frac{\Delta a}{\Delta F}$$

$$\text{slope} = \frac{4.8\ \text{m/s}^2}{12.0\ \text{N}}$$

$$\text{slope} = \frac{4.8\ \text{m/s}^2}{12.0\ \text{kg} \cdot \text{m/s}^2}$$

$$\text{slope} = \frac{1}{2.5\ \text{kg}}$$

Thus, $m = 2.5$ kg

Figure 2-11. The slope of the line on an acceleration-force graph gives the reciprocal of the mass of the object being accelerated. Note in the calculation that the newton, N, is equivalent to a kilogram · meter per second^2.

opposite in direction. The two equal and opposing forces constitute an action/reaction pair. The third law indicates that for every action force there is an equal and opposite reaction force. This means that a single force cannot be generated in nature. When one force is generated, another force of equal magnitude and opposite direction must also be generated.

SAMPLE PROBLEM

A 10.-newton force gives a mass m_1 an acceleration a. A 20.-newton force gives another mass m_2 the same acceleration a. What is the ratio of m_1 to m_2?

SOLUTION: Identify the known and unknown values.

Known

$F_1 = 10.\ \text{N}$

$F_2 = 20.\ \text{N}$

Unknown

$\frac{m_1}{m_2} = ?$

1. Write the formula that defines acceleration.

$$a = \frac{F_{net}}{m}$$

2. Solve the equation for m.

$$m = \frac{F_{net}}{a}$$

3. Substitute the known values for each mass.

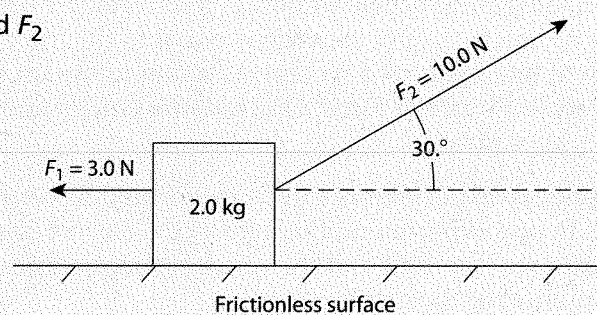
$$m_1 = \frac{10.\ \text{N}}{a} \text{ and } m_2 = \frac{20.\ \text{N}}{a}$$

4. Divide the first equation by the second.

$$\frac{m_1}{m_2} = \frac{10.\ \text{N}/a}{20.\ \text{N}/a} = \frac{1}{2}$$

SAMPLE PROBLEM

The diagram at the right shows two forces F_1 and F_2 applied to a 2.0-kilogram box originally at rest on a horizontal frictionless surface. Calculate the magnitude and direction of the acceleration of the box.



SOLUTION: Identify the known and unknown values.

Known

$m = 2.0\ \text{kg}$

Unknown

$a = ?\ \text{m/s}^2$ to the?

From the diagram:

$F_1 = 3.0\ \text{N}$ to the left
$F_2 = 10.0\ \text{N}$ to the right at 30.° above the horizontal

1. Write the formula for the horizontal component of a vector.

$$A_x = A \cos \theta$$

2. Substitute F_{2_x} for A_x, then substitute the known values and solve.

$$F_{2_x} = F_2 \cos \theta = (10.0\ \text{N})(\cos 30.°) = 8.7\ \text{N}$$

3. Determine the net force in the horizontal direction acting on the box.

$$F_{net_x} = F_{1_x} + F_{2_x} = -3.0\ \text{N} + 8.7\ \text{N} = 5.7\ \text{N}$$

4. Write the formula that defines Newton's second law.

$$a = \frac{F_{net}}{m}$$

5. Substitute the known values and solve.

$$a = \frac{5.7\ \text{N}}{2.0\ \text{kg}} = 2.9\ \text{m/s}^2 \text{ to the right}$$

A baseball bat striking a ball is an example of an action-reaction pair. If the bat exerts a 50.-newton force on the baseball, the ball exerts a 50.-newton force on the bat in the opposite direction. Each member of the action/reaction pair of forces acts on a different object; one force acts on the ball and the other on the bat. If no other forces are present, the objects are accelerated in opposite directions as long as the forces are applied.

Review Questions

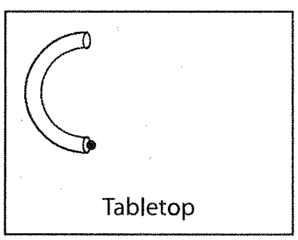
55. As the mass of an object on Earth's surface decreases, what happens to the inertia and weight of the object?

(1) Inertia decreases, and weight decreases.
(2) Inertia decreases, and weight remains the same.
(3) Inertia increases, and weight decreases.
(4) Inertia remains the same, and weight remains the same.

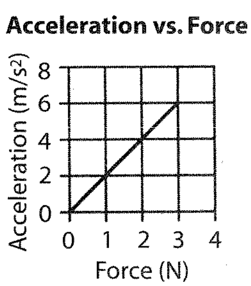
56. Compared to the inertia of a 0.10-kilogram steel ball, the inertia of a 0.20-kilogram Styrofoam ball is

(1) one-half as great (3) the same
(2) twice as great (4) four times as great

57. A ball rolls through a hollow semicircular tube lying flat on a horizontal tabletop. On the following diagram, draw a line with an arrowhead to represent the path of the ball after emerging from the tube, as viewed from above.



58. The following graph shows the relationship between the acceleration of an object and the net force on the object.



What is the mass of the object?

(1) 1 kg (2) 2 kg (3) 0.5 kg (4) 0.2 kg

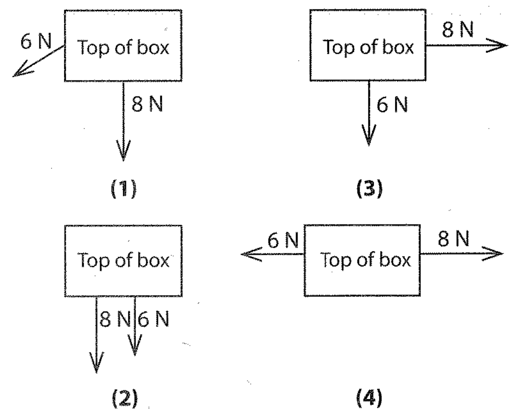
59. A cart is uniformly accelerating from rest. The net force acting on the cart is

(1) decreasing (3) constant but not zero
(2) zero (4) increasing

60. An 8.0-kilogram block and a 2.0-kilogram block rest on a horizontal frictionless surface. When horizontal force F is applied to the 8.0-kilogram block, it accelerates at 5.0 meters per second^2 east. If the same force was applied to the 2.0-kilogram block, the magnitude of the block's acceleration would be

(1) 1.3 m/s^2 (3) 10. m/s^2
(2) 2.5 m/s^2 (4) 20. m/s^2

61. A 6-newton force and an 8-newton force act concurrently on a box located on a frictionless horizontal surface. Which top-view diagram shows the forces producing the smallest magnitude of acceleration of the box?

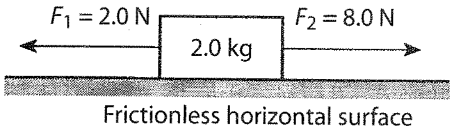


62. As the vector sum of all the forces acting on a moving object increases, the magnitude of the acceleration of the object

(1) decreases (3) remains the same
(2) increases

63. An unbalanced force of 10.0 newtons north acts on a 20.0-kilogram mass for 5.0 seconds. Calculate the acceleration of the mass.

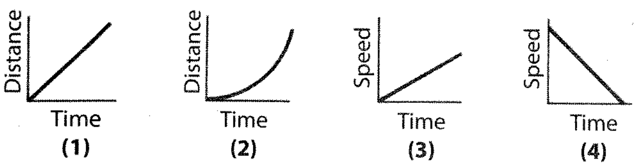
64 Two horizontal forces are applied to a 2.0-kilogram block on a frictionless, horizontal surface, as shown in the following diagram.



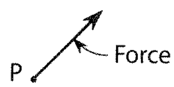
The acceleration of the block is

(1) 5.0 m/s^2 to the right
(2) 5.0 m/s^2 to the left
(3) 3.0 m/s^2 to the right
(4) 3.0 m/s^2 to the left

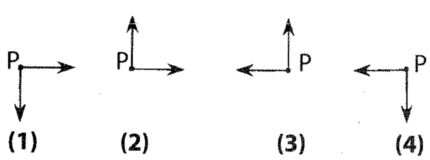
65. Which graph best represents the motion of an object on which the net force is zero?



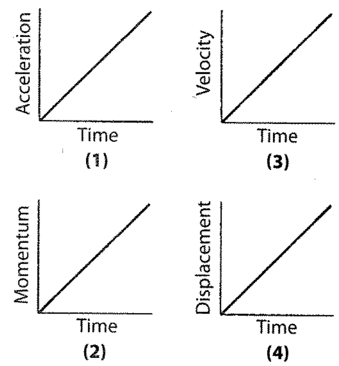
66. The vector diagram below represents force *F* acting on point *P*.



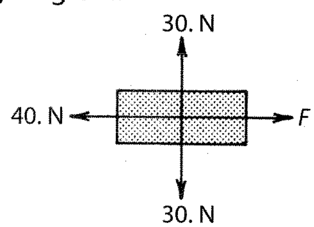
Which vector diagram represents the pair of concurrent forces that would produce equilibrium when added to force *F*?



67. Which graph best represents the motion of an object that has no unbalanced force acting on it?



68. Four forces act on an object, as shown in the following diagram.

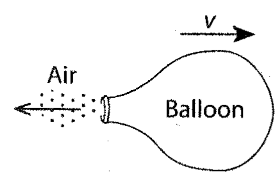


If the object is moving with a constant velocity, what is the magnitude of force *F*?

69. A 1.0-kilogram book rests on a horizontal tabletop. The magnitude of the force of the tabletop on the book is

(1) 1.0 kg (2) 9.8 kg (3) 1.0 N (4) 9.8 N

70. In the following diagram, an inflated balloon released from rest moves horizontally with velocity *v*.



What is the most likely cause of this velocity?

Two-Dimensional Motion and Trajectories

The motion of an object traveling in a two-dimensional plane can be described by separating its motion into the horizontal (x) and vertical (y) components of its displacement, velocity, and acceleration. A component parallel to the horizon is a **horizontal component** and a component at right angles to the horizon is a **vertical component.** If air resistance is neglected, an example of two-dimensional motion is the motion of a cannonball projected near the surface of Earth at an angle above the horizontal. If gravity is the only unbalanced force acting on the cannonball, the vertical component of the ball's motion is identical to that of a freely falling body, and the horizontal component is uniform motion. Although the two motions occur simultaneously, the two components of the motion are independent. Thus, if the object's initial velocity is known, the motion of the object in Earth's gravitational field can be described by the superposition of the two motions.

A Projectile Fired Horizontally

An object projected horizontally from some height above Earth's surface obeys Newton's laws of motion. If air resistance is neglected, the horizontal component of the velocity of the object remains constant. The initial vertical velocity of the object is zero but the vertical velocity increases as the object accelerates downward due to gravity. Figure 2-12A shows a ball falling freely straight downward from rest. Figure 2-12B shows a ball that has a horizontal component of velocity as it falls downward.

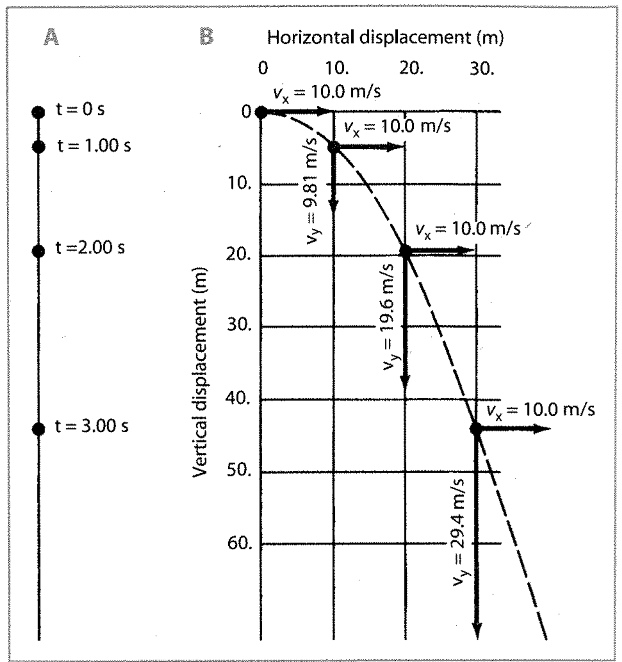


Figure 2-12. **(A) The position of a ball at 1.00-second intervals as it falls from rest in a vacuum near Earth's surface. The vertical scale in drawing B also applies to drawing A. (B) The position of the same ball at 1.00-second intervals after it has been rolled off the edge of a building with an initial horizontal velocity of 10.0 m/s. The arrows are velocity vectors giving the horizontal and vertical components of the velocity when the ball is at each position. Note that the horizontal component of the ball's velocity is the same after each second but the vertical component increases with time as the ball is accelerated by gravity. Note also that in both A and B, the vertical distance the ball has fallen at the end of each second is the same.**

Whether an object is dropped from rest or projected horizontally, the vertical distance fallen by the object is the same at any particular instant of time, as can be seen by comparing the vertical position of the ball at 1.00-second intervals in Figure 2-12A and B. This example illustrates that a ball thrown horizontally at 10.0 meters per second from a height of 44.1 meters above level ground, will hit the ground at the same time as another ball dropped from the same height at the same time.

In addition to showing the positions of the ball after an elapsed time of 1.00, 2.00, and 3.00 seconds, Figure 2-12B shows (by means of velocity vectors) the vertical and horizontal components of the velocity. At any particular time after the ball is released, the vertical component of velocity of an object projected horizontally is the same as the vertical component of velocity of an object dropped from rest. However, the vertical component of the ball's velocity increases, as the ball is accelerated by gravity. Thus, the vertical distance the ball falls in the third second is greater than the vertical distance the ball falls in the second second.

When air resistance is neglected, there is no acceleration or change in velocity in the horizontal direction. In Figure 2-12B, the horizontal displacement of the ball is a constant 10.0 meters in each 1.00-second interval. If the horizontal velocity of the ball had been greater, the object would have traveled a greater horizontal distance in the first 3.00 seconds of travel.

A Projectile Fired at an Angle

A golf ball is an example of an object that is projected with an initial velocity at an angle above the horizontal. Such a projectile rises to some height above Earth and then falls back to the ground. The projectile's motion can be studied by resolving the initial velocity into its horizontal and vertical components and then calculating the motions resulting from the two components. If air resistance is ignored, the horizontal component

SAMPLE PROBLEM

A plane flying horizontally at an altitude of 490 meters and having a velocity of 250 meters per second east, drops a supply packet to a work crew on the ground. It falls freely without a parachute. [Assume no wind and negligible air resistance.]
(a) Calculate the time required for the packet to hit the ground.
(b) Calculate the horizontal distance from the target area that the plane must drop the packet.

SOLUTION: Identify the known and unknown values.

Known	Unknown
$d_y = 490$ m	$t = ?$ s
$v_x = 250$ m/s	$d_x = ?$ m
$v_{i_y} = 0.0$ m/s	
$a_y = g = 9.81$ m/s^2	
$a_x = 0.00$ m/s^2	

1. Write a formula that relates the distance, acceleration, and time for motion in the vertical direction.

$$d_y = v_{i_y}t + \tfrac{1}{2}a_y t^2$$

Because v_{i_y} is zero, the equation becomes

$$d_y = \frac{1}{2}a_y t^2$$

2. Solve the equation for t and substitute g for acceleration.

$$t = \sqrt{\frac{2d_y}{g}}$$

3. Substitute the known values and solve.

$$t = \sqrt{\frac{2(490\ \text{m})}{9.81\ \text{m/s}^2}} = 10.\text{s}$$

4. Write a formula that relates the distance, acceleration, and time for motion in the horizontal direction.

$$d_x = v_{i_x}t + \tfrac{1}{2}a_x t^2$$

Because $a_x = 0.0$ m/s^2 the equation becomes $d_x = v_{i_x}t$.

5. Substitute the known values and solve.

$$d_x = (250\ \text{m/s})(10.\ \text{s}) = 2.5 \times 10^3\ \text{m}$$

The plane must drop the packet 2.5×10^3 meters west of the target.

of the velocity remains constant. The object's vertical motion is accelerated by the force of gravity.

If a golf ball is projected with initial velocity v_i at an angle θ above the horizontal, v_i can be separated into perpendicular components, as shown in Figure 2-13.

Recall from page 36 that any vector, A, making an angle θ with the horizontal can be resolved into horizontal and vertical components. The two components of velocity can be determined using these formulas.

Horizontal component: $v_{i_x} = v_i \cos\theta$
Vertical component: $v_{i_y} = v_i \sin\theta$

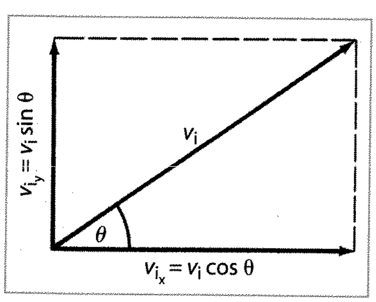


Figure 2-13. An initial velocity vector resolved into horizontal and vertical components

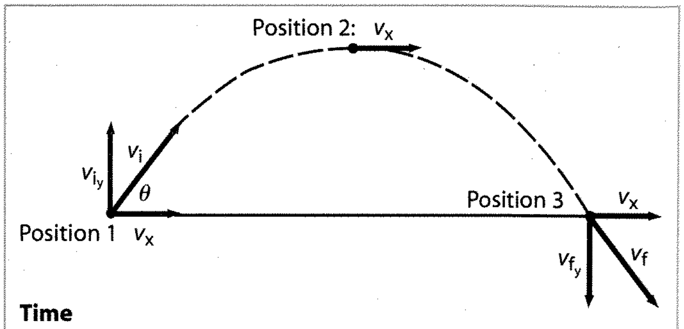
The vertical component of the velocity gradually decreases to zero as the golf ball reaches the highest point in its trajectory. When the vertical component of the velocity is zero, all of the velocity is in the horizontal. Then the vertical component gradually increases along the ball's downward path due to the constant acceleration of gravity. See Figure 2-14.

To find the time t for the projectile to reach its maximum height, solve the equation $v_f = v_i + at$ for t.

$$t = \frac{v_f - v_i}{a}$$

Then, substitute the appropriate values for the vertical velocity and acceleration. If upward in the vertical direction is considered positive, then g, a downward acceleration, is negative. At the highest point, the vertical velocity of the projectile is zero. Thus $t = \frac{v_i \sin\theta}{g}$. It can be shown that the time for the projectile to reach its maximum height is the same as the time to fall back to the ground from that height. Therefore the total time of travel to return to ground level is $\frac{2v_i \sin\theta}{g}$.

The horizontal distance traveled by a projectile is called its range. For any given initial velocity, the range is a maximum when $\theta = 45°$. For any given initial speed of a projectile, the range of the projectile is the same for complementary angles above the horizontal, neglecting friction. For example, a projectile launched with an initial velocity of 15 meters per second at 20.° above the horizontal has the same range as a projectile launched with an initial velocity of 15 meters per second at 70.° above the horizontal. However, the time of flight and maximum height above the horizontal are greater for the launch at 70.°. The actual range of a projectile (when air resistance is present) is shorter than the calculated ideal range.



Time

Position 1:	Position 2:	Position 3:
$t = 0$	$t = \frac{v_i \sin\theta}{g}$	$t = \frac{2 v_i \sin\theta}{g}$

Vertical motion

Position 1:	Position 2:	Position 3:
$v_{i_y} = v_i \sin\theta$	$v_y = 0$	$v_{f_y} = -v_i \sin\theta$
$a_y = -g$	$a_y = -g$	$a_y = -g$

Horizontal motion

Position 1:	Position 2:	Position 3:
	$v_x = v_i \cos\theta$ in all positions	
	$a_x = 0$ in all positions	

Figure 2-14. The motion of a projectile that is fired at angle θ above the horizontal and returns to the horizontal. [Neglect friction.]

SAMPLE PROBLEM

A small missile is fired with a velocity of 300. meters per second at an angle of 30.0° above the ground. After a total flight time of 30.6 seconds, the missile returns to the level ground. [Neglect air resistance.]
(a) Calculate the initial horizontal and vertical components of the velocity.
(b) Calculate the maximum height of the missile above the ground.
(c) Calculate the total horizontal range of the missile.

SOLUTION: Identify the known and unknown values.

Known	*Unknown*
$v_i = 300.$ m/s	$v_{i_x} = ?$ m/s
$\theta = 30.0°$	$v_{i_y} = ?$ m/s
$t = 30.6$ s	$d_y = ?$ m
$a_y = g = -9.81$ m/s^2	$d_x = ?$ m
$a_x = 0.00$ m/s^2	

1. Write the formulas that resolve the initial velocity vector into horizontal and vertical components.

$v_{i_x} = v_i \cos\theta$
$v_{i_y} = v_i \sin\theta$

2. Substitute the known values and solve.

$v_{i_x} = (300.\text{ m/s})(\cos 30.0°) = 260.\text{ m/s}$
$v_{i_y} = (300.\text{ m/s})(\sin 30.0°) = 150.\text{ m/s}$

3. Find the time for the missile to reach its highest point.

Because the total time of flight is 30.6 s, the time to reach the maximum height is $\frac{1}{2}(30.6\text{ s}) = 15.3$ s.

4. Write the formula that relates distance, average velocity, and time in the vertical direction.

$d_y = \bar{v}_y t_{rise}$

At the highest point, velocity in the vertical direction is zero. Average velocity = $\frac{1}{2}(v_i + v_f) = \frac{1}{2}(150.\text{ m/s} + 0\text{ m/s}) = 75.0$ m/s.

5. Substitute the known values and solve.
$d_y = (75.0 \text{ m/s})(15.3 \text{ s}) = 1150 \text{ m}$

- An alternate solution is to write the formula that relates displacement, time, and acceleration, $d = v_i t + \frac{1}{2}at^2$ and substitute the known values for the vertical direction. That is

$d_y = (150. \text{ m/s})(15.3 \text{ s}) + \frac{1}{2}(-9.81 \text{ m/s}^2)(15.3 \text{ s})^2$
$d_y = 1150 \text{ m}$

- Another alternate solution is to write the formula that relates final velocity, initial velocity, acceleration, and displacement, $v_f^2 = v_i^2 + 2ad$. Solve the equation for d to yield $d = \frac{v_f^2 - v_i^2}{2a}$. Then substitute the known values for the vertical direction. That is $d = \frac{(0 \text{ m/s})^2 - (150. \text{ m/s})^2}{2(-9.81 \text{ m/s}^2)} = 1150 \text{ m}$

6. Write the equation that relates distance, velocity, and time in the horizontal direction.
$d_x = \bar{v}_x t_{total}$

7. Substitute the known values and solve.
$d_x = (260. \text{ m/s})(30.6 \text{ s}) = 7960 \text{ m}$

Review Questions

Base your answers to questions 71 through 75 on the following information.

A ball of mass *m* is thrown horizontally with speed *v* from a height *h* above level ground. [Neglect friction.]

71. If the height above the ground from which the ball is thrown was increased, the time of flight of the ball would

(1) decrease (2) increase (3) remain the same

72. If the initial speed of the ball was increased, the time of flight of the ball would

(1) decrease (2) increase (3) remain the same

73. If the initial speed of the ball was increased, the horizontal distance traveled by the ball would

(1) decrease (2) increase (3) remain the same

74. As time elapses before the ball strikes the ground, the horizontal velocity of the ball

(1) decreases (3) remains the same
(2) increases

75. Compared to the total horizontal distance traveled by the ball in the absence of air resistance, the total horizontal distance traveled by the ball with air resistance is

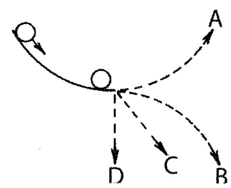
(1) shorter (2) longer (3) the same

---

76. A student throws a baseball horizontally at 25 meters per second from a cliff 45 meters above the level ground. Approximately how far from the base of the cliff does the ball hit the ground? [Neglect air resistance.]

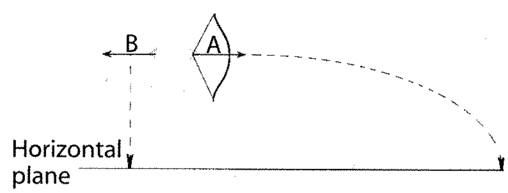
(1) 45 m (2) 75 m (3) 140 m (4) 230 m

77. A ball rolls down a curved ramp, as shown in the following diagram.



Which dotted line best represents the path of the ball after leaving the ramp?

78. Above a flat horizontal plane, arrow *A* is shot horizontally from a bow at a speed of 20 meters per second, as shown in the following diagram. A second arrow *B* is dropped from the same height and at the same instant as *A* is fired.



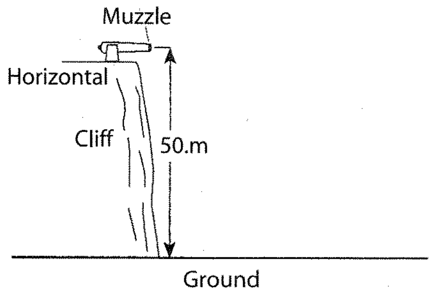
Compare the amount of time *A* takes to strike the plane to the amount of time *B* takes to strike the plane. [Neglect friction.]

79. A rock is thrown horizontally from the top of a cliff at 12 meters per second. Calculate the time required for the rock to fall 45 meters vertically.

80. A ball is thrown horizontally at a speed of 24 meters per second from the top of a cliff. If the ball hits the ground 4.0 seconds later, approximately how high is the cliff?

(1) 6.0 m (2) 39 m (3) 78 m (4) 96 m

81. The following diagram shows the muzzle of a cannon located 50. meters above the ground. When the cannon is fired, a ball leaves the muzzle with an initial speed of 250 meters per second. [Neglect air resistance].



Which action would most likely increase the time of flight of a ball fired by the cannon?

(1) pointing the muzzle of the cannon toward the ground
(2) moving the cannon closer to the edge of the cliff
(3) positioning the cannon higher above the ground
(4) giving the ball a greater initial horizontal velocity

82. A football player kicks a ball with an initial velocity of 25 meters per second at an angle of 53° above the horizontal. The vertical component of the initial velocity of the ball is

(1) 25 m/s (2) 20. m/s (3) 15 m/s (4) 10. m/s

83. The path of a projectile fired at an angle of 30.° above the horizontal is best described as

(1) parabolic (3) circular
(2) linear (4) hyperbolic

84. A projectile is fired with velocity of 150. meters per second at an angle of 30.° above the horizontal. Calculate the magnitude of the horizontal component of the velocity at the time the projectile is fired.

85. Projectile *A* is fired with velocity *v* at an angle of 30.° above the horizontal. Projectile *B* is fired with velocity *v* at an angle of 40.° above the horizontal. Compared to the magnitude of the horizontal component of *v* at the time projectile *A* is fired, the magnitude of the horizontal component of *v* at the time projectile *B* is fired is

(1) smaller (2) larger (3) the same

86. A projectile is launched at an angle of 60.° above the horizontal. Compared to the vertical component of the initial velocity of the projectile, the vertical component of the projectile's velocity when it has reached its maximum height is

(1) smaller (2) larger (3) the same

87. A projectile is launched at an angle of 30.° above the horizontal. Neglecting air resistance, what are the projectile's horizontal and vertical accelerations when it reaches its maximum height?

Uniform Circular Motion

According to Newton's first law of motion, an unbalanced force acting on an object always produces a change in the object's velocity. If the force has a component in the direction of the object's motion, the magnitude of the velocity changes. However, if the force is applied perpendicular to the direction of motion, only the direction of the velocity changes; its magnitude remains the same. In both instances, the object accelerates because velocity changes with time. If the applied force has a constant magnitude and always acts perpendicular to the direction of the object's velocity, the object moves in a circular path at constant speed, experiencing **uniform circular motion.**

Centripetal Acceleration

An object moving uniformly in a circular path always has **centripetal acceleration,** which is an acceleration directed toward the center of the circle. "Center-seeking" centripetal acceleration is a vector quantity whose magnitude is directly proportional to the square of the speed of the object and inversely proportional to the radius of the circular path in which it travels. Centripetal acceleration is represented by this formula.

$$a_c = \frac{v^2}{r}$$ (R)

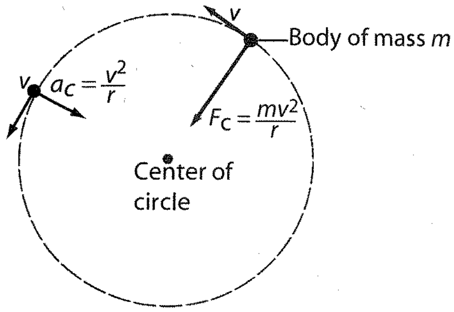


Figure 2-15. The relationship between velocity *v*, mass *m*, radius of curvature *r*, centripetal force F_c, and centripetal acceleration a_c for a body in uniform circular motion: The velocity vector is tangent to the circle. Both the centripetal force and the centripetal acceleration are directed toward the center of the circle. The radius of curvature is the radius of the circle.

The speed v of the object is in meters per second, the radius of curvature r is in meters, and the centripetal acceleration a_c is in meters per second^2. The centripetal acceleration of an object is independent of the mass of the object.

Centripetal Force

The force needed to keep an object moving in a circular path is called **centripetal force,** F_c. Centripetal force is a force directed toward the center of curvature. Centripetal force, a vector quantity, produces centripetal acceleration. Newton's second law, $F = ma$, can be rewritten for the special case of circular motion as $F_c = ma_c$. Substituting in the expression for centripetal acceleration, $a_c = \frac{v^2}{r}$ yields this formula.

$$F_c = \frac{mv^2}{r}$$

Mass m is in kilograms, speed v is in meters per second, radius r is in meters, and centripetal force F_c is in kilogram · meters per second^2, or newtons. Figure 2-15 shows the relationship between these quantities for an object in uniform circular motion.

An object in uniform circular motion travels at constant speed in its circular path because there is no net force acting on the object in its direction of motion. That is, the object's tangential velocity is constant. However, the object is not in equilibrium because centripetal force acts perpendicular to the tangential velocity and produces a constant acceleration towards the center of curvature. Thus, although the magnitude of the object's velocity (speed) remains constant, the direction of the object's velocity is always changing, as shown in Figure 2-16.

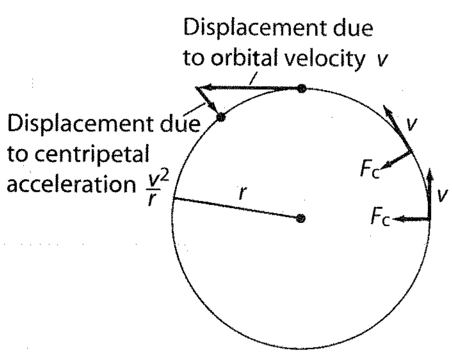


Figure 2-16. The velocity, acceleration, and displacement of a body in uniform circular motion: The velocity vector is always tangent to the circle and perpendicular to the centripetal acceleration. The acceleration causes a continuous change in the direction of the velocity and a continuous displacement to the circular path.

SAMPLE PROBLEM

A 1.5-kilogram cart moves in a horizontal circular path of 1.3-meter radius at a constant speed of 2.0 meters per second.
(a) Calculate the magnitude of the centripetal acceleration of the cart.
(b) Calculate the magnitude of the centripetal force on the cart.

SOLUTION: Identify the known and unknown values.

Known	Unknown
$m = 1.5$ kg	$a_c = ?$ m/s^2
$r = 1.3$ m	$F_c = ?$ N
$v = 2.0$ m/s	

1. Write the formula for centripetal acceleration.

$$a_c = \frac{v^2}{r}$$

2. Substitute the known values and solve.

$$a_c = \frac{(2.0 \text{ m/s})^2}{1.3 \text{ m}} = \frac{4.0 \text{ m}^2/\text{s}^2}{1.3 \text{ m}} = 3.1 \text{ m/s}^2$$

3. Write the formula for centripetal force.

$$F_c = \frac{mv^2}{r}$$

4. Substitute the known values and solve.

$$F_c = \frac{(1.5 \text{ kg})(2.0 \text{ m/s})^2}{1.3 \text{ m}} = \frac{(1.5 \text{ kg})(4.0 \text{ m}^2/\text{s}^2)}{1.3 \text{ m}}$$

$F_c = 4.6$ N

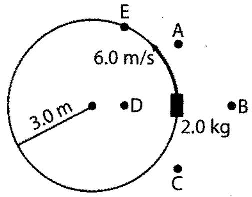
Another way to solve (b) is to substitute the calculated value of a_c from part (a) for v^2/r.

$F_c = ma_c = (1.5 \text{ kg})(3.1 \text{ m/s}^2) = 4.7$ N

Review Questions

Base your answers to questions 88 through 96 on the following information and diagram.

A 2.0-kilogram cart travels counter-clockwise at a constant speed of 6.0 meters per second in a horizontal circle of radius 3.0 meters.



88. Calculate the magnitude and direction of the centripetal acceleration of the cart at the position shown.

89. Calculate the magnitude of the centripetal force acting on the cart.

90. If the mass of the cart was doubled, the magnitude of the centripetal force acting on the cart would be

(1) halved
(2) doubled
(3) quartered
(4) quadrupled

91. If the radius of curvature of the path was doubled, the magnitude of the centripetal acceleration of the cart would be

(1) halved
(2) doubled
(3) quartered
(4) quadrupled

92. If the speed of the cart was doubled, the magnitude of the centripetal force on the cart would be

(1) halved
(2) doubled
(3) quartered
(4) quadrupled

93. If the mass of the cart was halved, the magnitude of the centripetal acceleration of the cart would

(1) decrease (2) increase (3) remain the same

94. In the position shown in the diagram, towards which point is the centripetal force acting on the cart directed?

95. In the position shown in the diagram, towards which point is the velocity of the cart directed?

96. Which factor, when doubled, would produce the greatest change in the magnitude of the centripetal force acting on the cart?

(1) mass of the cart
(2) radius of curvature of the path
(3) speed of the cart
(4) weight of the cart

97. As the time taken for a car to make one lap around a circular track decreases, the centripetal acceleration of the car

(1) decreases (2) increases (3) remains the same

98. The tangential acceleration of a cart moving at a constant speed in a horizontal circle is

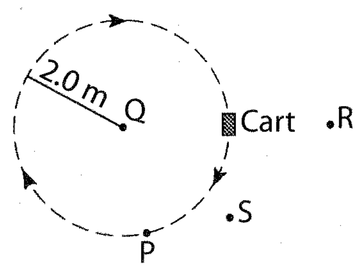
(1) 0.0 m/s^2
(2) 9.8 m/s^2 in the direction of the velocity
(3) constant in magnitude and directed radially toward the center of curvature
(4) constant in magnitude and directed radially away from the center of curvature

99. The centripetal acceleration of a ball of mass m moving at constant speed v in a horizontal circular path of radius r is

(1) zero
(2) constant in direction, but changing in magnitude
(3) constant in magnitude, but changing in direction
(4) changing in both magnitude and direction

Base your answers to questions 100 through 103 on the following information and diagram.

A 5.0-kilogram cart travels clockwise in a horizontal circle of radius 2.0 meters at a constant speed of 4.0 meters per second



100. Towards which point is the velocity of the cart directed at the position shown?

101. Towards which point is the centripetal acceleration of the cart directed at the position shown?

102. If the mass of the cart was doubled, the magnitude of the cart's centripetal acceleration would be

(1) unchanged
(2) doubled
(3) halved
(4) quadrupled

103. The magnitude of the centripetal force acting on the cart is

(1) 8.0 N (2) 20. N (3) 40. N (4) 50. N

Newton's Universal Law of Gravitation

Every body in the universe exerts a force of attraction on every other body. According to Newton's universal law of gravitation, any two bodies attract each other with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers. The attractive force that one object exerts on another object due to their masses is called **gravitational force,** which is given by this formula.

$$F_g = \frac{Gm_1m_2}{r^2}$$ Ⓡ

In the formula F_g is the gravitational force in newtons, m_1 and m_2 are the masses of the objects in kilograms, r is the distance between the centers of the objects in meters, and G is the universal gravitational constant $6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$. The universal law of gravitation is valid only for spherical masses of uniform density and masses that are small compared to the distance between their centers.

According to the law, the gravitational force that mass m_1 exerts on mass m_2 is equal in magnitude and opposite in direction to the gravitational force that mass m_2 exerts on mass m_1. If the distance between the centers of the two masses is doubled, the magnitude of the gravitational force is quartered. If one of the two masses is doubled and the distance between their centers remains constant, the magnitude of the gravitational force is doubled.

SAMPLE PROBLEM

Calculate the magnitude of the gravitational force of attraction that Earth exerts on the Moon.

SOLUTION: Identify the known and unknown values. Obtain needed values from the *Reference Tables for Physical Setting/Physics.*

Known

$m_{Earth} = 5.98 \times 10^{24}$ kg

$m_{Moon} = 7.35 \times 10^{22}$ kg

$r_{Earth\ to\ Moon} = 3.84 \times 10^{8}$ m

$G = 6.67 \times 10^{-11}\ \text{N} \bullet \text{m}^2/\text{kg}^2$

Unknown

$F_g = ?$ N

1. Write the formula for the gravitational force.

$$F_g = \frac{Gm_1m_2}{r^2}$$

2. Substitute the known values and solve.

$$F_g = \frac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\ \text{kg})(7.35 \times 10^{22}\ \text{kg})}{(3.84 \times 10^{8}\ \text{m})^2}$$

$$F_g = 1.99 \times 10^{20}\ \text{N}$$

Gravitational Field Strength A region in space where a test particle would experience a gravitational force is called a **gravitational field.** Every mass is surrounded by a gravitational field. A unit test mass is used to map a gravitational field, such as the one that surrounds Earth.

Figure 2-17A shows gravitational force vectors associated with a test mass at various locations above Earth's surface. The direction of the vectors indicates that the test mass is always attracted to Earth, and the magnitude of the vectors indicates that the force on the test mass increases as it gets closer to Earth. In Figure 2-17B, the force vectors have been joined to form lines of gravitational force. The imaginary line along which a test mass would move in a gravitational field is called a line of gravitational force.

In Figure 2-17 the gravitational field lines are directed radially toward the center of Earth, that is, normal to Earth's surface. The concentration of the field lines increases as Earth's surface is approached. This indicates that gravitational field strength, a vector quantity, increases as the distance from Earth decreases. At any point in a gravitational field, **gravitational field strength,** g, equals the force per unit mass at that point. The relationship is expressed by this formula.

$$g = \frac{F_g}{m}$$ Ⓡ

On a mass m in kilograms the gravitational force F_g is in newtons, and the gravitational field strength g is in newtons per kilogram, N/kg. Gravitational field strength has the same direction as the gravitational force acting on the mass.

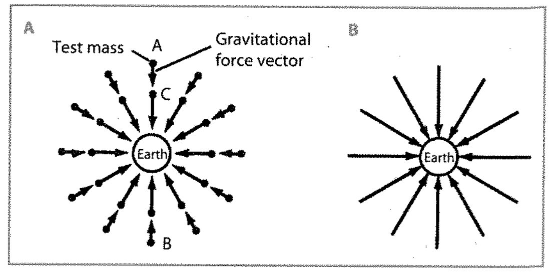


Figure 2-17. The gravitational field around Earth: (A) When the test mass is at points *A* or *B*, the magnitude of the gravitational force is the same because both points are the same distance from the center of Earth. At point *C* the gravitational force is greater than at points *A* and *B* because *C* is closer to the center of Earth. (B) The force vectors have been joined to form lines of gravitational force.

The unit for gravitational field strength is the same as the unit for acceleration. Because 1 newton = 1 kilogram · meter / second^2, then

$$1\,\frac{\text{newton}}{\text{kilogram}} = 1\,\frac{\cancel{\text{kilogram}}\cdot\text{meter / second}^2}{\cancel{\text{kilogram}}}$$
$$= 1\text{ meter / second}^2$$

Recall that the acceleration of an object equals the ratio $\frac{F_{net}}{m}$ from the equation $a = \frac{F_{net}}{m}$. Consequently g is the acceleration produced on a mass m by the gravitational force F_g. Therefore, the gravitational field strength g is the same as the acceleration due to gravity. For short distances near the surface of Earth, the gravitational field is considered to be uniform and g is the same for all masses:

g (gravitational field strength) = 9.81 N/kg

g (acceleration due to gravity) = 9.81 m/s^2

Do not confuse g the acceleration due to gravity (9.81 m/s^2) with G the universal gravitational constant (6.67×10^{-11} N · m^2/kg^2).

Weight

The gravitational force with which a planet attracts a mass is called **weight.** If M is the mass of Earth, m is the mass of an object on Earth's surface, and r is the distance from the center of Earth, it can be seen from Newton's universal law of gravitation that the weight F_g of an object on Earth's surface is directly proportional to its mass m because all the other quantities in the equation are constant. Weight is a vector quantity (force) directed toward the center of a planet that is measured in newtons, whereas mass is a scalar quantity measured in kilograms. The weight of an object decreases with increasing distance from the center of a planet because the gravitational field strength decreases. But the mass of an object is constant because it is independent of its location in any gravitational field.

The weight of an object can be determined by solving the gravitational field strength equation for F_g ($F_g = mg$) and substituting values for the mass and the acceleration due to gravity. The result shows that the weight F_g of a 1.00-kilogram object on Earth's surface is 9.81 newtons. The weight

Static and Kinetic Friction

There are several kinds of friction. **Static friction** is the force that opposes the start of motion, whereas **kinetic friction** is the friction between objects in contact when they are in motion. Once motion starts, kinetic friction decreases. The force of kinetic friction for two surfaces in contact is less than the force of static friction for the same two surfaces, so the coefficient of kinetic friction is less than the coefficient of static friction. For example, according to the *Reference Tables for Physical Setting/Physics*, the coefficient of kinetic friction for copper on steel is 0.36 and the coefficient of static friction for copper on steel is 0.53.

Figure 2-19 shows forces acting concurrently on a 10.0-newton wooden block in equilibrium on a wooden horizontal surface. In each case, the normal force is equal in magnitude and opposite in direction to the weight of the block. In Figure 2-19A, the applied horizontal force is equal in magnitude but opposite in direction to the maximum static friction force. In Figure 2-19B, the horizontal force applied to move the block at constant speed to the right is equal in magnitude but opposite in direction to the force of kinetic friction. When the block is moved to the right at constant speed, the net force acting on the block in the horizontal direction is zero, and the block is in equilibrium.

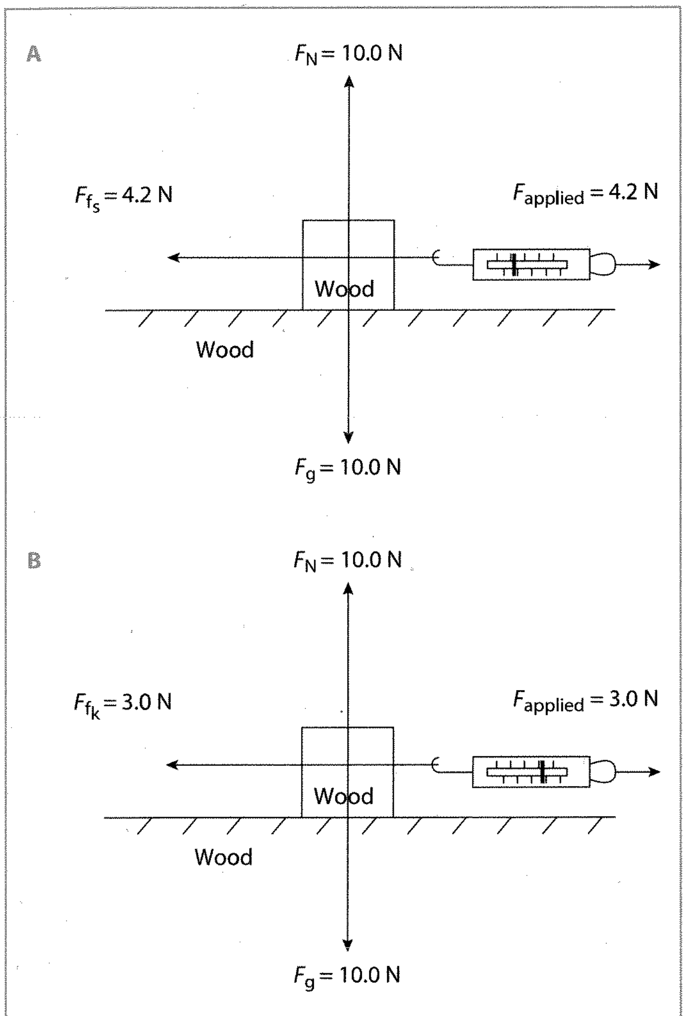


Figure 2-19. A horizontal force is applied to a 10.0-newton wooden block on a horizontal wooden surface: (A) A maximum static friction force keeps the box from moving. (B) The box moves at constant speed to the right when the applied force equals the force of kinetic friction. Note: The vectors are *not* drawn to scale.

Using the information in Figure 2-19, the coefficients of static and kinetic friction are

$$\mu_s = \frac{F_{f_s}}{F_N} = \frac{4.2 \text{ N}}{10.0 \text{ N}} = 0.42$$

and

$$\mu_k = \frac{F_{f_k}}{F_N} = \frac{3.0 \text{ N}}{10.0 \text{ N}} = 0.30$$

The values agree with those found in the *Reference Tables for Physical Setting/Physics*.

Determining the Coefficient of Friction A graph of frictional force versus normal force (weight) for a wooden block in contact with a wooden horizontal surface is a straight line for both static friction and kinetic friction. (Experimentally, the weight of the block can be varied by resting masses on top of it, thus keeping the nature of the two surfaces in contact the same at all times.) The slopes of the lines are the coefficient of static friction and the coefficient of kinetic friction, respectively. Figure 2-20 shows the lines that would result for data collected for a wooden block on a wooden table. The slope of the static friction line is 0.42 and the slope of the kinetic friction line is 0.30.

Friction on an Inclined Surface If an object is on a surface inclined at angle θ to the horizontal, the object's weight can be resolved into two components, one perpendicular to the inclined surface and the other parallel to the surface. The perpendicular component of the object's weight, $F_g \cos\theta$ or $mg \cos\theta$, is equal in magnitude and opposite in direction to the normal force and has no effect on the motion of the object. The object cannot move in the direction of either force. Only the component of the object's weight parallel to the inclined surface $F_g \sin\theta$ or $mg \sin\theta$ tends to accelerate the object down the incline. As the angle that the incline makes with the horizontal increases, the component of the object's weight parallel to the incline increases, and the acceleration of the object down the incline increases. This acceleration is opposed by the friction between the object and the incline. The magnitude of the force of friction is directly proportional the normal force, which is equal in magnitude but opposite in direction to the perpendicular component of the object's weight. Thus, as the angle of inclination increases, the component of the objects's weight perpendicular to the incline decreases, and the magnitude of the frictional force decreases. The steeper the slope of the incline, the greater the acceleration of the object down the incline.

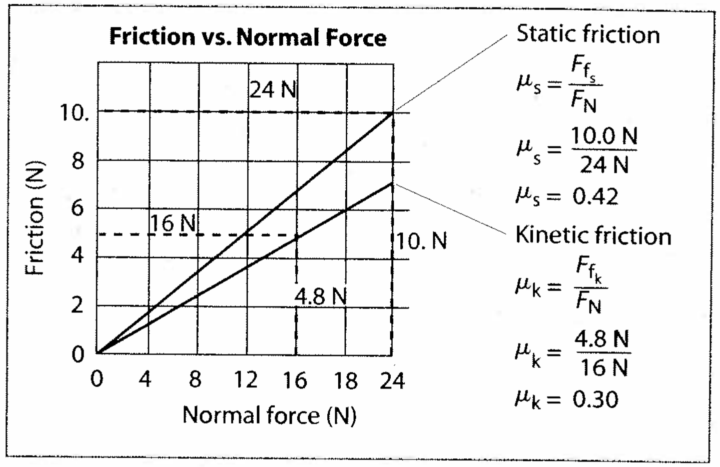


Figure 2-20. Finding the coefficients of static and kinetic friction by determining the slopes of the lines on a friction-normal force graph

Fluid Friction Fluid friction, which results from an object moving through a fluid such as air depends upon the surface area and the speed of the object moving through the fluid.

Review Questions

116. An empty wooden crate slides across a warehouse floor. If the crate was filled, the coefficient of kinetic friction between the crate and the floor would

(1) decrease (2) increase (3) remain the same

117. An empty wooden crate slides across a warehouse floor. If the crate was filled, the magnitude of the force of kinetic friction between the crate and the floor would

(1) decrease (2) increase (3) remain the same

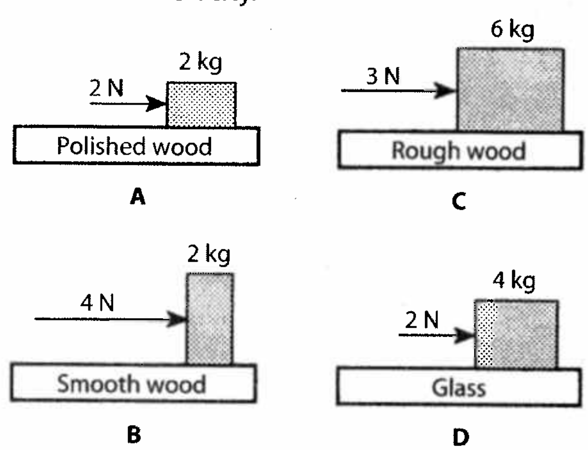
118. As an object initially at rest on a horizontal surface is set in motion, the magnitude of the force of friction between the object and the surface

(1) decreases (2) increases (3) remains the same

119. As a thrown baseball is acted on by air friction, the thermal energy of the ball

(1) decreases (2) increases (3) remains the same

120. Each of the following diagrams shows a different block being pushed to the right by a horizontal force across a horizontal surface at constant velocity.



In which two diagrams is the force of friction the same?

Base your answers to questions 121 to 123 on the information below.

A force of 10. newtons toward the right is exerted on a wooden crate initially moving to the right on a horizontal wooden floor. The crate weighs 25 newtons.

121. Calculate the magnitude of the force of friction between the crate and the floor.

122. What is the magnitude of the net force acting on the crate?

123. Is the crate accelerating? Explain your answer.

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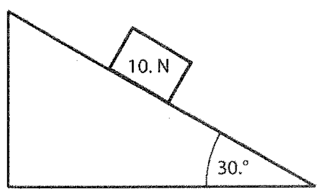
124. The diagram below represents a 10.-newton block sliding at constant speed down a plane that makes an angle of 30.° with the horizontal.



The magnitude of the frictional force acting on the block is

(1) 5.0 N (2) 10. N (3) 49 N (4) 98 N

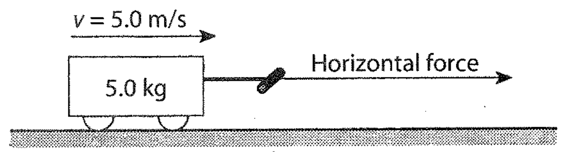
125. Sand is often placed on an icy road because the sand

(1) decreases the coefficient of friction between the tires of a car and the road
(2) increases the coefficient of friction between the tires of a car and the road
(3) decreases the gravitational force on a car
(4) increases the normal force of a car on the road

126. A 10.-kilogram rubber block is pulled horizontally at constant velocity across a sheet of ice. Calculate the magnitude of the force of friction acting on the block.

Base your answers to questions 127 through 130 on the following information and diagram.

A horizontal force is used to pull a 5.0-kilogram cart at a constant speed of 5.0 meters per second across the floor. The force of friction between the cart and the floor is 10. newtons.



127. What is the magnitude of the horizontal force along the handle of the cart?

128. Calculate the weight of the cart.

129. Compare the weight of the cart to the normal force.

130. Calculate the coefficient of kinetic friction between the cart and the floor.

---

131. A constant horizontal force of 5.2 newtons is applied to a wooden block to slide it at constant speed across a wooden table. Calculate the weight of the block.

132. A wooden block is at rest on a wooden inclined plane. As the angle the plane makes with the horizontal is increased, the coefficient of static friction between the block and the plane

(1) decreases
(2) increases
(3) remains the same

133. A wooden block is at rest on a wooden inclined plane. As the angle the plane makes with the horizontal is increased, the magnitude of the force of static friction between the block and the plane

(1) decreases
(2) increases
(3) remains the same

Momentum

The product of an object's mass and velocity is a vector quantity called **momentum.** It is given by this formula.

$$p = mv$$

Mass m is in kilograms, velocity v is in meters per second, and momentum p is in kilogram · meters per second. The direction of an object's momentum is the same as the direction of its velocity. The SI unit for momentum is kilogram · meters per second, kg · m/s.

Impulse and Change in Momentum

The product of the net force acting on an object and the time during which the force acts is called **impulse.** Impulse, a vector quantity having the same direction as the net force, is given by this formula.

$$J = F_{net}t$$ (R)

The average force F is in newtons, t is the time during which the force acts in seconds, and J is the impulse in newton · seconds. The SI unit for impulse is the newton · second, N · s.

The impulse imparted to an object can also be determined graphically. A horizontal force of varying magnitude is applied over time to an object on a horizontal surface and a graph of force versus time is plotted. The area under the line equals the impulse imparted to the object.

According to Newton's second law an unbalanced force acting on an object causes it to accelerate. This acceleration produces a change in the object's velocity and consequently its momentum, as shown by the following equations.

$$F_{net} = ma = m\frac{\Delta v}{t} \text{ or } F_{net}t = m\Delta v$$

Because $F_{net}t$ equals the impulse and $m\Delta v$ equals the change in momentum, it follows that

$$J = F_{net}t = \Delta p$$ (R)

The direction of the impulse imparted to an object is the same as the direction of the object's change in momentum. If an object is in equilibrium, there is no change in its momentum and, thus, no impulse imparted to it.

SAMPLE PROBLEM

A 5.0-kilogram object has an initial velocity of 8.0 meters per second due east. An unbalanced force acts on the object for 3.0 seconds, causing its velocity to decrease to 2.0 meters per second east. Calculate the magnitude and direction of the unbalanced force.

SOLUTION: Identify the known and unknown values.

Known	Unknown
$m = 5.0$ kg	F_{net} = ? N to the ?
$v_i = 8.0$ m/s east	
$v_f = 2.0$ m/s east	
$t = 3.0$ s	

1. Write the formula relating impulse and change in momentum and the formula for momentum.

$J = F_{net}t = \Delta p$ and $p = mv$

2. Combine the formulas.

$F_{net}t = m\Delta v$

3. Solve the equation for F.

$$F_{net} = \frac{m\Delta v}{t} = \frac{m(v_f - v_i)}{t}$$

4. Substitute the known values and solve.

$$F_{net} = \frac{(5.0\text{ kg})(2.0\text{ m/s} - 8.0\text{ m/s})}{3.0\text{ s}} = \frac{-30.\text{ kg}\cdot\text{m/s}}{3.0\text{ s}}$$

$F_{net} = -10.\text{ kg} \bullet \text{m/s}^2 = -10.\text{ N}$

The force is 10. N directed to the west.

In baseball, both the batter hitting the ball and the outfielder catching the ball are aware of the relationship between impulse and momentum. The batter "follows through" to keep the bat in contact with the ball as long as possible. The greater the time during which the force of impact acts on the ball, the larger the impulse imparted to it, the greater its final momentum, and the longer the distance of travel. The outfielder catching the ball tries to prolong the time of slowing the ball by moving the gloved hand back in the direction of the ball's motion. By increasing the time during which the gloved hand acts on the ball to reduce its momentum to zero, the force needed to produce the necessary impulse and the "sting" are reduced.

Conservation of Momentum

A group of objects, not acted upon by any external force, is called a **closed system.** According to Newton's third law, within such a system the force F exerted by one mass m_1 in the system on a second mass m_2 must be equal in magnitude and opposite in direction to the force that m_2 exerts on m_1. Because the force F acts on both masses for exactly the same amount of time, the magnitude of the impulse on each mass is the same. Consequently, the change in momentum for each mass has the same magnitude, but they are in opposite directions. The relationship is expressed in this way.

$$m_1\Delta v_1 = -m_2\Delta v_2, \text{ or } m_1\Delta v_1 + m_2\Delta v_2 = 0$$

The total change in momentum due to the interaction of masses m_1 and m_2 is zero. This relationship is summed up in the **law of conservation of momentum** which states that the total momentum of the objects in a closed system is constant. The law is given by this formula.

(R)

$$p_{before} = p_{after}$$

Momentum p is in kilogram • meters per second, kg • m/s.

SAMPLE PROBLEM

A 1.0-kilogram cart A is initially at rest on a horizontal frictionless air track. A 0.20-kilogram cart B is moving to the right at 10.0 meters per second on the same track. Cart B collides with cart A causing cart A to move to the right at 3.0 meters per second. Calculate the velocity of cart B after the collision.

SOLUTION: Identify the known and unknown values. Let velocity to the right be positive.

Known	Unknown
$m_A = 1.0$ kg	$v_{B_f} =$? m/s to the ?
$m_B = 0.20$ kg	
$v_{A_i} = 0.0$ m/s	
$v_{B_i} = 10.0$ m/s	
$v_{A_f} = 3.0$ m/s	

1. Write the formula that equates the momentum of the system before and after the collision.

$p_{after} = p_{before}$

$m_A v_{A_f} + m_B v_{B_f} = m_A v_{A_i} + m_B v_{B_i}$

2. Solve the equation for v_{B_f}.

$$v_{B_f} = \frac{m_A v_{A_i} + m_B v_{B_i} - m_A v_{A_f}}{m_B}$$

3. Substitute the known values and solve.

$$v_{B_f} = \frac{(1.0 \text{ kg})(0.0 \text{ m/s}) + (0.20 \text{ kg})(10.0 \text{ m/s}) - (1.0 \text{ kg})(3.0 \text{ m/s})}{0.20 \text{ kg}}$$

$v_{B_f} = -5.0$ m/s

The velocity of cart B after the collision is 5.0 m/s to the left.

Review Questions

134. As an object falls freely toward Earth, the object's momentum

(1) decreases (2) increases (3) remains the same

135. Which term identifies a scalar quantity?

(1) acceleration
(2) momentum
(3) speed
(4) displacement

136. What is the magnitude of the velocity of a 25-kilogram object that has a momentum of 100. kilogram · meters per second?

(1) 0.25 m/s
(2) 2500 m/s
(3) 40. m/s
(4) 4.0 m/s

137. What is the momentum of a 1,200-kilogram car traveling at 15 meters per second due east?

(1) 80. kg · m/s due east
(2) 80. kg · m/s due west
(3) 1.8×10^4 kg · m/s due east
(4) 1.8×10^4 kg · m/s due west

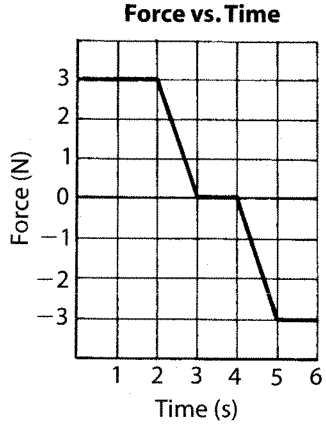
138. A constant unbalanced force acts on an object for 3.0 seconds, producing an impulse of 6.0 newton • seconds east. Determine the magnitude and direction of the force.

139. A 10.-newton force to the east acts on an object for 0.010 second. What force to the east, acting on the object for 0.050 second, would produce the same impulse?

(1) 1.0 N (2) 2.0 N (3) 5.0 N (4) 10. N

Base your answers to questions 140 and 141 on the graph below, which represents the relationship between the net force acting on an object and the time during which the force acts.

Force vs. Time



140. During which time interval is the velocity of the object constant?

141. Determine the net impulse imparted to the object during the 6-second time interval.

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142. What is the magnitude of the net force acting on a 2.0×10^3-kilogram car as it accelerates from rest to a speed of 15 meters per second in 5.0 seconds?

(1) 6.0×10^3 N
(2) 2.0×10^4 N
(3) 3.0×10^4 N
(4) 6.0×10^4 N

143. A 5.00-kilogram mass is traveling at 100. meters per second. Determine the speed of the mass after an impulse with a magnitude of 30.0 newton • seconds is applied.

144. A 2,400-kilogram car is traveling at a speed of 20. meters per second. Compared to the magnitude of the force required to stop the car in 12 seconds, the magnitude of the force required to stop the car in 6.0 seconds is

(1) half as great
(2) twice as great
(3) the same
(4) four times as great

145. A 2.0-kilogram cart moving due east at 6.0 meters per second collides with a 3.0-kilogram cart moving due west. The carts stick together and come to rest after the collision. Calculate the initial speed of the 3.0-kilogram cart.

146. A 0.180-kilogram cart traveling at 0.80 meter per second to the right collides with a 0.100-kilogram cart initially at rest. The carts lock together upon collision. Calculate the final velocity of the carts.

147. A 2.0-kilogram cart traveling north at 4.0 meters per second collides head on with a 1.0-kilogram cart traveling south at 8.0 meters per second. What is the magnitude of the total momentum of the two carts immediately after the collision?

(1) 0.0 kg · m/s
(2) 8.0 kg · m/s
(3) 16 kg · m/s
(4) 32 kg · m/s

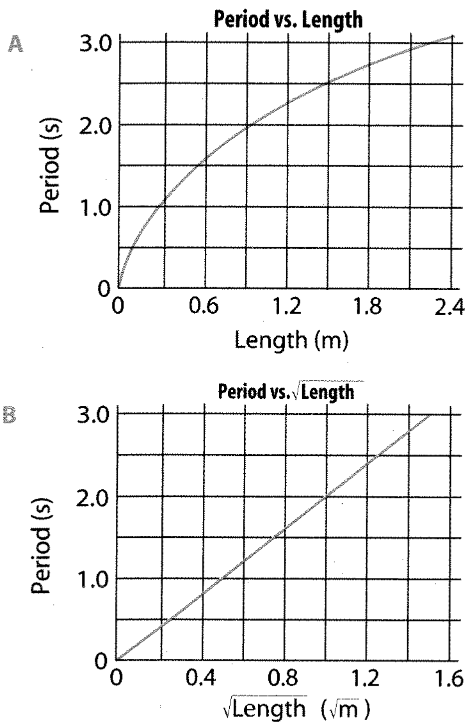


Figure 2-21. Graphs of experimental data of the period versus the length and square root of length of a simple pendulum: (A) shows that the period is not directly proportional to the length. (B) confirms the equation that defines the period of a pendulum. The period is directly proportional to the square root of the length.

The Simple Pendulum

A simple **pendulum** consists of a bob or mass m attached to a string of negligible mass. The length of the pendulum ℓ is measured from the pivot point at one end of the string to the center of the bob, where all the mass is assumed to be concentrated. In the equilibrium position, the string is perpendicular to the ground. To set the pendulum in motion, the bob is displaced from the equilibrium position by lifting it in the gravitational field. The angle the string makes with the equilibrium position is called the amplitude, θ.

Period of a Simple Pendulum

If friction is negligible, the variables associated with a simple pendulum are mass, length, amplitude, and gravitational field strength. The time required for the displaced pendulum to complete one cycle of motion is called the **period,** T. The number of cycles the pendulum completes per unit time is called the frequency, f. The period of the pendulum is related to the frequency by the equation $T = \frac{1}{f}$ where period is in seconds and frequency is in hertz (Hz), or 1/s.

It can be found experimentally that for amplitudes less than 15°, the period of a simple pendulum is independent of the mass of the bob, and independent of the amplitude. However, the period is affected by the length of the pendulum ℓ and the acceleration due to gravity g. The period is given by this formula.

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

The length l is in meters, the acceleration due to gravity g is in meters per second^2, and the period T is in seconds. **Note to student:** This formula does not appear on the *Reference Tables for Physical Setting/Physics*, and is not testable. However, plotting given data for the period of a simple pendulum versus length or the period versus the square root of length and interpreting the resulting graphs is testable. Figure 2-21 shows two graphs produced as a result of varying the length of a simple pendulum and measuring its period.

Equilibrium and Nonequilibrium Forces

When a pendulum is in the equilibrium position, two forces act on the bob, the weight F_g and the tension in the string F_T. The tension is equal in magnitude and opposite in direction to the weight, so there is no net force on the bob. If the bob is displaced from equilibrium and the pendulum has an amplitude θ, the pendulum is no longer in equilibrium. The tension in the string is still directed along the string, but it is not opposite in direction to the weight. If the weight of the bob is resolved into perpendicular components F_{g_x} and F_{g_y}, the tension in the string is found to be less than the weight of the bob. The net force on the bob, equal to the component of its weight, F_{g_x}, acts along the tangent to its path. This net force causes the bob to accelerate towards its equilibrium position.

Practice Questions for the New York Regents Exam

Directions

Review the Test-Taking Strategies section of this book. Then answer the following questions. Read each question carefully and answer with a correct choice or response.

Part A

1 A student walks 1.0 kilometer due east and 1.0 kilometer due south. Then she runs 2.0 kilometers due west. The magnitude of the student's resultant displacement is

(1) 0 km (2) 1.4 km (3) 3.4 km (4) 4.0 km

2 A car travels 20. meters east in 1.0 second. The displacement of the car at the end of this 1.0-second interval is

(1) 20. m (3) 20. m east
(2) 20. m/s (4) 20. m/s east

3 Two cars, A and B, are 400. meters apart. Car A travels due east at 30. meters per second on a collision course with car B, which travels due west at 20. meters per second. What is the total time that elapses before the two cars collide?

(1) 8.0 s (2) 13 s (3) 20. s (4) 40. s

4 A baseball pitcher throws a fastball at 42 meters per second. If the batter is 18 meters from the pitcher, approximately how much time does it take for the ball to reach the batter?

(1) 2.3 s (2) 1.9 s (3) 0.86 s (4) 0.43 s

5 The velocity of an object in linear motion changes from +25 meters per second to +15 meters per second in 2.0 seconds. What is the object's acceleration during this 2.0-second interval?

(1) $-20.\ m/s^2$ (3) $-5.0\ m/s^2$
(2) $+20.\ m/s^2$ (4) $+5.0\ m/s^2$

6 An object initially traveling in a straight line with a velocity of 5.0 meters per second north is accelerated uniformly at 2.0 meters per second^2 north for 4.0 seconds. What is the total distance traveled by the object in this 4.0-second interval?

(1) 36 m (2) 24 m (3) 16 m (4) 4.0 m

7 An object initially at rest accelerates uniformly in a straight line at 5.0 meters per second^2, until it attains a speed of 30. meters per second. What is the total distance the object moves while accelerating?

(1) 180 m (2) 150 m (3) 3.0 m (4) 90. m

8 A stone is dropped from a bridge 45 meters above the surface of a river. What is the time required for the stone to reach the water's surface? [Neglect friction.]

(1) 9.2 s (2) 4.6 s (3) 3.0 s (4) 0.22 s

9 A ball is thrown straight up with a speed of 12 meters per second near the surface of Earth. What is the maximum height reached by the ball? [Neglect friction.]

(1) 15 m (2) 7.3 m (3) 1.2 m (4) 0.37 m

10 An object falls freely from rest near the surface of Earth. What is the speed of the object when it has fallen 4.9 meters from its rest position?

(1) 4.9 m/s (2) 9.8 m/s (3) 24 m/s (4) 96 m/s

11 Starting from rest, object A falls freely for 2.0 seconds and object B falls freely for 4.0 seconds. Compared with the distance fallen by object A, the distance fallen by object B is

(1) half as far (3) three times as far
(2) twice as far (4) four times as far

12 Two concurrent forces have a maximum resultant of 45 newtons and a minimum resultant of 5.0 newtons. What is the magnitude of each of these forces?

(1) 0.0 N and 45 N (3) 20. N and 25 N
(2) 5.0 N and 9.0 N (4) 0.0 N and 50. N

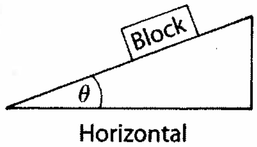
13 Two forces act concurrently on an object. The magnitude of the resultant will be greatest when the angle between the forces is

(1) 0° (2) 60.° (3) 90.° (4) 180.°

14 Two concurrent forces act at right angles to each other on an object. If the magnitude of one of the forces is 40. newtons and the magnitude of the resultant of the two forces is 50. newtons, the magnitude of the other force must be

(1) 10. N (2) 20. N (3) 30. N (4) 40. N

15 In the diagram below, a block is at rest on a plane inclined at an angle θ to the horizontal.



As angle θ is increased, what happens to the magnitude of the component of the block's weight parallel to the plane and the magnitude of the component of the block's weight perpendicular to the plane?

(1) Both components decrease.
(2) The parallel component decreases and the perpendicular component increases.
(3) The parallel component increases and the perpendicular component decreases.
(4) Both components increase.

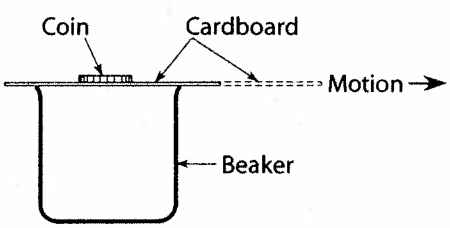
16 Compared to 8 kilograms of feathers, 6 kilograms of lead has

(1) less mass and less inertia
(2) less mass and more inertia
(3) more mass and less inertia
(4) more mass and more inertia

17 A box, initially at rest on a level floor, is being acted upon by a horizontal force. Compared to the force required to start the box moving, the force required to keep the box moving at constant speed is

(1) smaller (2) greater (3) the same

18 A copper coin rests on a piece of cardboard placed on a beaker as shown in the following diagram.



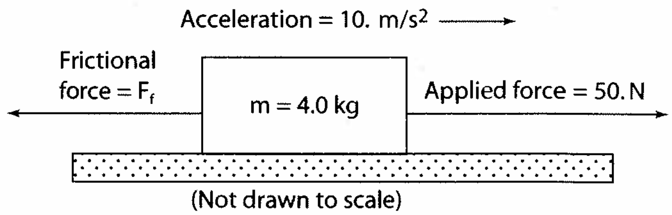
Which two properties of the coin best explain why the coin falls into the beaker when the cardboard is rapidly removed?

(1) weight and volume
(2) weight and inertia
(3) electrical resistance and volume
(4) electrical resistance and inertia

19 What is the magnitude of the force required to give an electron an acceleration of 1.00×10^2 meters per second^2?

(1) 9.11×10^{-33} N
(2) 9.11×10^{-31} N
(3) 9.11×10^{-29} N
(4) 1.10×10^{32} N

20 The diagram below shows a 4.0-kilogram object accelerating at 10. meters per second^2 on a rough horizontal surface.



What is the magnitude of the frictional force F_f acting on the object?

(1) 5.0 N (2) 10. N (3) 20. N (4) 40. N

21 An object is moving on a horizontal frictionless surface. If the net force applied to the object in the direction of motion is doubled, the magnitude of the acceleration of the object is

(1) halved
(2) doubled
(3) unchanged
(4) quadrupled

22 In which situation is the net force on the object equal to zero?

(1) a satellite moving at constant speed around Earth in a circular orbit
(2) an automobile braking to a stop
(3) a bicycle moving at constant speed on a straight, level road
(4) a pitched baseball being hit by a bat

23 Equilibrium exists in a system where three forces are acting concurrently on an object. If the system includes a 5.0-newton force due north and a 2.0-newton force due south, the third force must be

(1) 7.0 N south
(2) 7.0 N north
(3) 3.0 N south
(4) 3.0 N north

24 A baseball bat moving at high speed strikes a feather. If air resistance is neglected, compared to the magnitude of the force exerted by the bat on the feather, the magnitude of the force exerted by the feather on the bat is

(1) smaller (2) larger (3) the same

25 A baseball player throws a ball horizontally. Which statement best describes the ball's motion after it is thrown? [Neglect friction.]
(1) Its vertical speed remains the same and its horizontal speed increases.
(2) Its vertical speed remains the same and its horizontal speed remains the same.
(3) Its vertical speed increases and its horizontal speed increases.
(4) Its vertical speed increases and its horizontal speed remains the same.

26 A red ball and a green ball are simultaneously thrown horizontally from the same height. The red ball has an initial speed of 40. meters per second and the green ball has an initial speed of 20. meters per second. Compared to the time it takes the red ball to reach the ground, the time it takes the green ball to reach the ground is
(1) the same (3) half as much
(2) twice as much (4) four times as much

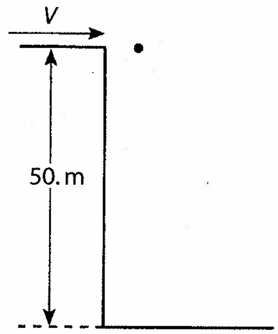
27 A student throws a stone upward at an angle of 45° above the horizontal. Which statement best describes the stone at the highest point that it reaches?
(1) Its acceleration is zero.
(2) Its acceleration is a minimum, but not zero.
(3) Its gravitational potential energy is a minimum.
(4) Its kinetic energy is a minimum.

28 A projectile is launched with an initial velocity of 200 meters per second at an angle of 30° above the horizontal. What is the magnitude of the vertical component of the projectile's initial velocity?
(1) $200 \text{ m/s} \times \cos 30°$ (3) $\frac{200 \text{ m/s}}{\cos 30°}$
(2) $200 \text{ m/s} \times \sin 30°$ (4) $\frac{200 \text{ m/s}}{\sin 30°}$

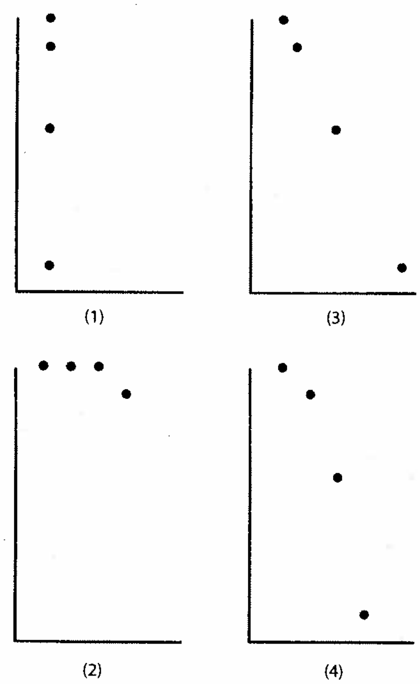
29 An amusement park ride moves a rider at a constant speed of 14 meters per second in a horizontal circular path of radius 10. meters. What is the magnitude of the rider's centripetal acceleration in terms of g, the acceleration due to gravity?
(1) $1g$ (2) $2g$ (3) $3g$ (4) $0g$

30 As a cart travels around a horizontal circular track, the cart must undergo a change in
(1) velocity (3) speed
(2) inertia (4) weight

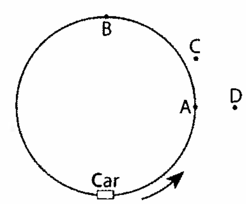
31 A ball is projected horizontally to the right from a height of 50. meters, as shown in the following diagram.



Which diagram best represents the position of the ball at 1.0-second intervals?

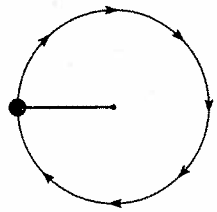


32 A convertible car with its top down is traveling at constant speed around a horizontal circular track, as shown in the following diagram.

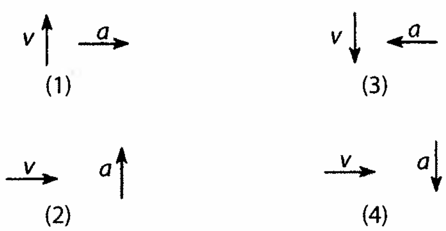


When the car is at point A, if a passenger in the car throws a ball straight up, the ball could land at point
(1) A (2) B (3) C (4) D

33 The diagram below represents a ball attached to one end of a string undergoing uniform circular motion as it travels clockwise.



At the moment shown in the diagram, what are the directions of both the ball's velocity and centripetal acceleration?



34 A satellite of mass m orbits Earth in a circular path of radius R. If centripetal force F_c is acting on the satellite, its speed is equal to

(1) $\sqrt{\frac{F_cR}{m}}$ (3) $\sqrt{\frac{F_cm}{R}}$

(2) $\frac{F_cR}{m}$ (4) F_cmR

35 If the mass of one of two particles is doubled and the distance between their centers is doubled, the magnitude of the force of attraction of one particle for the other particle is

(1) halved (3) quartered

(2) doubled (4) unchanged

36 What is the magnitude of the gravitational force that one 5.0-kilogram mass exerts on another 5.0-kilogram mass when the distance between their centers is 5.0 meters?

(1) 5.0×10^0 N (3) 6.7×10^{-11} N

(2) 3.3×10^{-10} N (4) 1.3×10^{-11} N

37 Two point masses m_1 and m_2 are located distance D apart. The magnitude of the gravitational force that m_1 exerts on m_2 can be quadrupled by changing the distance to

(1) $\frac{1}{2}D$ (2) $2D$ (3) $\frac{1}{4}D$ (4) $4D$

38 A 50-kilogram student, standing on Earth, attracts Earth with a force having a magnitude of

(1) 0 N (3) 5×10^1 N

(2) 5 N (4) 5×10^2 N

39 A 2.00-kilogram object weighs 19.6 newtons on Earth. If the acceleration due to gravity on Mars is 3.71 meters per second^2, what is the object's mass on Mars?

(1) 2.64 kg (3) 19.6 N

(2) 2.00 kg (4) 7.42 N

40 Which combination of units can be used to express the weight of an object?

(1) kilogram / second

(2) kilogram • meter

(3) kilogram • meter / second

(4) kilogram • meter / second^2

41 A 15-kilogram mass weighs 60. newtons on planet X. The mass is allowed to fall freely from rest near the surface of the planet. After falling for 6.0 seconds, the acceleration of the mass is

(1) 0.25 m/s^2 (3) 24 m/s^2

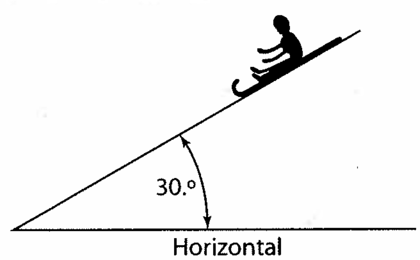
(2) 10. m/s^2 (4) 4.0 m/s^2

42 An object is allowed to fall freely from rest near the surface of a planet. If the object falls 54 meters in the first 3.0 seconds after it is released, what is the acceleration due to gravity on the planet?

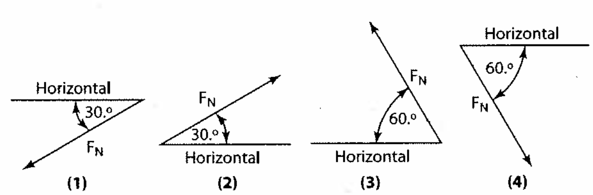
(1) 6.0 m/s^2 (3) 12 m/s^2

(2) 9.8 m/s^2 (4) 18 m/s^2

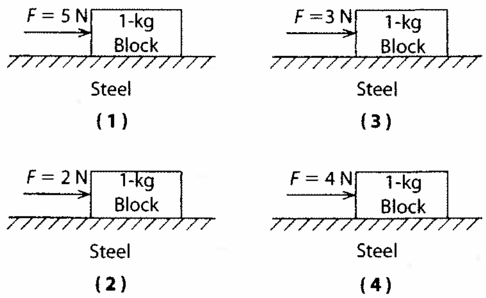
43 The diagram below shows a sled and rider sliding down a snow-covered hill that makes an angle of 30.° with the horizontal.



Which vector best represents the direction of the normal force, F_N, exerted by the hill on the sled?



44 A different horizontal force is applied to each of four 1-kilogram blocks to slide them across a uniform horizontal steel surface at constant speed as shown. In which diagram is the coefficient of friction between the block and steel smallest?



45 The magnitude of the momentum of an object is 64.0 kilogram • meters per second. If the magnitude of the velocity of the object is doubled, the magnitude of the object's momentum could be
(1) 32.0 kg • m/s
(2) 64.0 kg • m/s
(3) 128 kg • m/s
(4) 256 kg • m/s

46 A force of 20. newtons to the left is exerted on a cart for 10. seconds. For what period of time must a 50.-newton force to the right act to produce an impulse of equal magnitude on the cart?
(1) 10. s
(2) 2.0 s
(3) 5.0 s
(4) 4.0 s

47 A bullet traveling at 5.0×10^2 meters per second is brought to rest by a 50. newton • second impulse. What is the mass of the bullet?
(1) 2.5×10^4 kg
(2) 1.0×10^1 kg
(3) 1.0×10^{-1} kg
(4) 1.0×10^{-2} kg

48 A 5.0-kilogram cart traveling at 4.0 meters per second is brought to rest in 2.0 seconds. The magnitude of the average force used to stop the cart is
(1) 2.5 N (3) 20. N
(2) 10. N (4) 40. N

49 Two cars having different weights are traveling on a level surface at different constant velocities. Within the same time interval, greater force will always be required to stop the car that has the greater
(1) weight (3) velocity
(2) kinetic energy (4) momentum

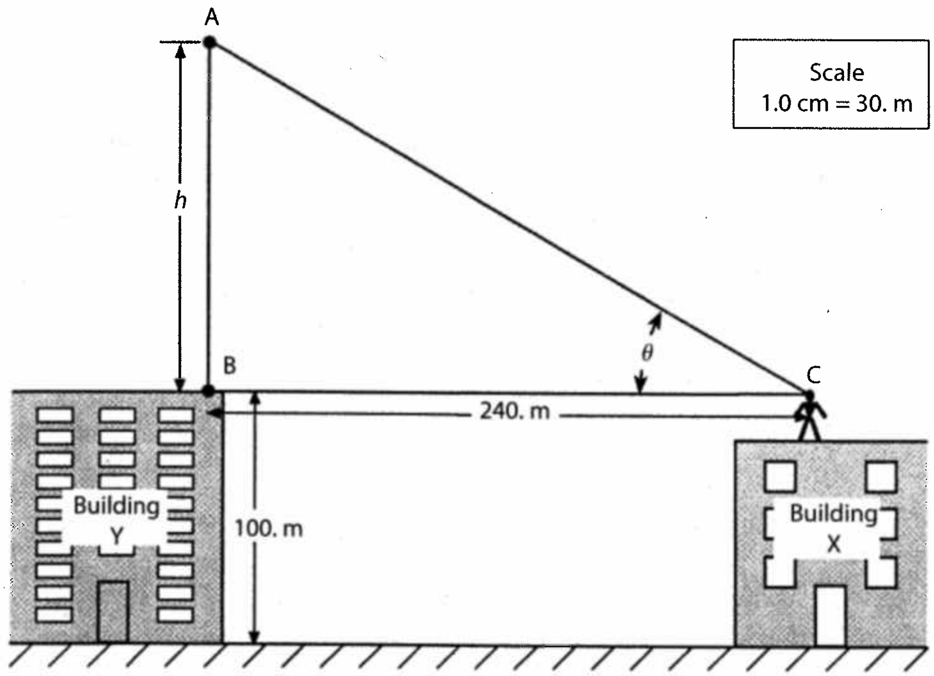
50 A 0.050-kilogram bullet is fired from a 4.0-kilogram rifle which is initially at rest. If the bullet leaves the rifle with momentum having a magnitude of 20. kilogram • meters per second, the rifle will recoil with a momentum having a magnitude of
(1) 1600 kg • m/s (3) 20. kg • m/s
(2) 80. kg • m/s (4) 0.25 kg • m/s

51 A 2.0-kilogram toy cannon is at rest on a frictionless horizontal surface. A remote triggering device causes a 0.005-kilogram projectile to be fired from the cannon. Which equation describes the system after the cannon is fired?
(1) mass of cannon + mass of projectile = 0
(2) speed of cannon + speed of projectile = 0
(3) momentum of cannon + momentum of projectile = 0
(4) velocity of cannon + velocity of projectile = 0

52 Which pair of terms are vector quantities?
(1) force and mass
(2) distance and displacement
(3) acceleration and momentum
(4) velocity and speed

Part B

Base your answers to questions 53 through 58 on the following diagram and information. The diagram is drawn to a scale of 1.0 centimeter = 30. meters.



A student on building X is located 240. meters from launch site B of a rocket on building Y. The rocket reaches its maximum altitude at point A. The student's eyes are level with the launch site on building Y. [Neglect friction.]

53 Using a protractor, measure the angle of elevation θ of the rocket. [1]

54 Determine the height h of the rocket above the student's eye level. [1]

55 Determine the total distance the rocket must fall from its maximum altitude to reach the ground. [1]

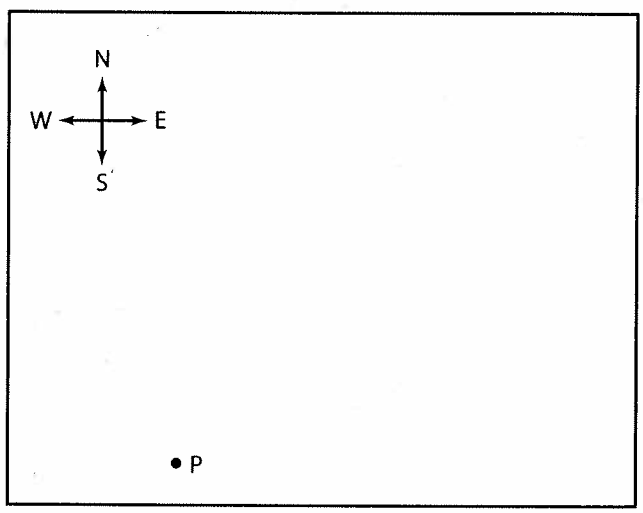
56 Calculate the total time required for the rocket to fall freely from point A back to ground level. [2]

57 Calculate the speed of the rocket as it reaches the ground after falling freely from point A. [2]

58 Sketch a graph to represent the relationship between velocity v and time t for the rocket from the time it is launched until it hits the ground. [1]

Base your answers to questions 59 through 62 on the information that follows.

A newspaper carrier on her delivery route travels 200. meters due north and then turns and walks 300. meters due east.



59 Using a ruler and a protractor and starting at point P, construct the sequence of two displacement vectors for the newspaper carrier's route. Use a scale of 1.0 centimeter = 50. meters. Label the vectors. [3]

60 Construct and label the vector that represents the carrier's resultant displacement from point P. [1]

61 Determine the magnitude of the carrier's resultant displacement. [1]

62 Using a protractor, measure the angle between north and the carrier's resultant displacement. [1]

Base your answers to questions 63 through 70 on the information that follows.

A stone is thrown with an initial velocity of 20. meters per second straight upward from the edge of a cliff 100. meters above a canyon floor. The stone just misses the cliff's edge on its way down. [Neglect friction.]

63 Calculate the time required for the stone to reach its maximum height. [2]

64 Calculate the maximum height of the stone above the edge of the cliff. [2]

65 Calculate the total time that elapses as the stone falls from its maximum height to the level from which it was thrown. [1]

66 What is the velocity of the stone upon returning to the level from which it was thrown? [1]

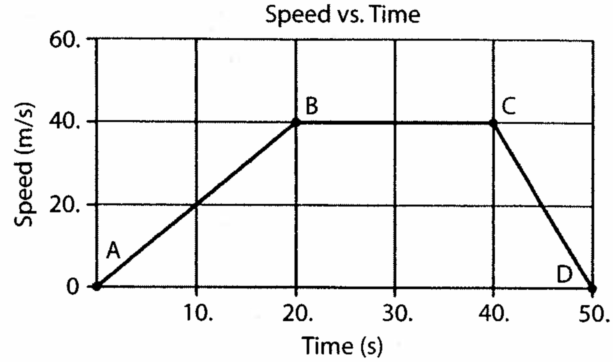
67 Calculate the velocity of the stone 6.0 seconds after it is thrown. [2]

68 Calculate the position of the stone 6.0 seconds after it is thrown. [2]

69 Sketch a graph to show the relationship between the stone's velocity and elapsed time from 0.0 second to 6.0 seconds. [1]

70 Sketch a graph to show the relationship between the stone's speed and elapsed time from 0.0 second to 6.0 seconds. [1]

Base your answers to questions 71 through 75 on the following speed-time graph, which represents the linear motion of a cart.



71 Calculate the magnitude of the acceleration of the cart during interval AB. [2]

72 Calculate the total distance traveled by the cart during interval ABC. [2]

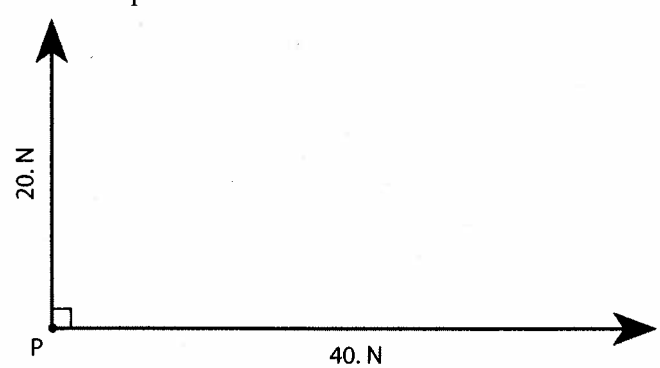
73 Determine the average speed of the cart during interval CD. [1]

74 Describe the motion of the cart during interval CD. [1]

75 Identify the interval during which the net force acting on the cart is zero. [1]

Base your answers to questions 76 through 80 on the following information and vector diagram.

A 20.-newton force due north and a 40.-newton force due east act concurrently on a 10.-kilogram object located at point P.



76 Use a ruler to determine the scale used in the vector diagram by finding the number of newtons represented by each centimeter. [1]

77 On the vector diagram, use a ruler and a protractor to construct the vector that represents the resultant force. [1]

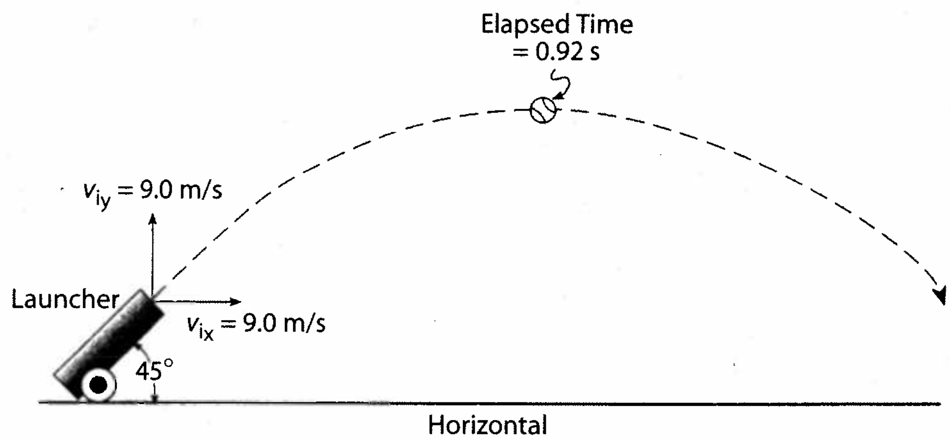
78 Determine the magnitude of the resultant force. [1]

79 Using a protractor measure the angle between east and the resultant vector. [1]

80 Calculate the magnitude of the acceleration of the object. [2]

Base your answers to questions 81 through 87 on the information and diagram below.

A machine launches a tennis ball at an angle of 45° with the horizontal, as shown. The ball has an initial vertical velocity of 9.0 meters per second and an initial horizontal velocity of 9.0 meters per second. The ball reaches its maximum height 0.92 seconds after its launch. [Neglect friction and assume the ball lands at the same height from which it was launched.]

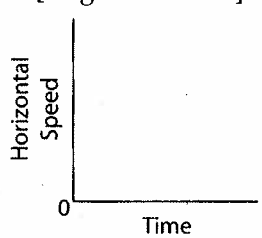


81 Calculate the speed of the ball as it leaves the launcher. [2]

82 Calculate the total horizontal distance traveled by the ball during the entire time it is in the air. [2]

83 Compare the vertical acceleration of the ball at the time of launch to the vertical acceleration of the ball at elapsed time 0.92 second. [1]

84 Sketch a graph to represent the relationship between the horizontal speed of the ball and elapsed time. [Neglect friction.]



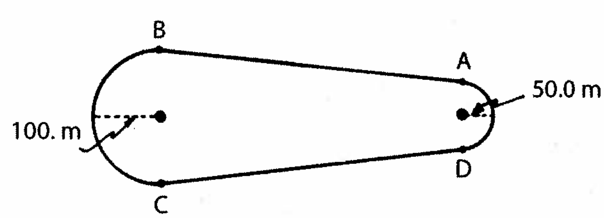
85 State the change, if any, in both the vertical component and horizontal component of the ball's velocity as it rises. [1]

86 On the diagram draw an arrow to show the direction of the ball's velocity at its maximum height. Label the arrow v. [1]

87 On the diagram draw an arrow to show the direction of the ball's acceleration at its maximum height. Label the arrow a. [1]

Base your answers to questions 88 through 93 on the following information and diagram.

A flat racetrack viewed from above has curves with radii of 50.0 meters and 100. meters. A car having a mass of 1.00×10^3 kilograms moves counterclockwise around the track at a constant speed of 20.0 meters per second. It takes the car 20.0 seconds to travel from C to D.



88 What is the magnitude of the net force acting on the car while it is moving from A to B? [1]

89 Calculate the net force acting on the car while it is moving from B to C. [2]

90 Calculate the distance from C to D. [2]

91 Compare the magnitude of the centripetal acceleration of the car while moving from D to A, to the magnitude of the centripetal acceleration of the car while moving from B to C. [1]

92 Compare the magnitude of the car's momentum at D to the magnitude of the car's momentum at B. [1]

93 Compare the magnitude of the centripetal acceleration of the car at A to the magnitude of the car's centripetal acceleration at A if additional passengers were riding in the car. [1]

Base your answers to questions 94 through 98 on the following information and data table.

An astronaut on a distant planet conducted an experiment to determine the gravitational acceleration on that planet. The data table shows the results of the experiment.

94 On the grid provided mark an appropriate scale on the axis labeled "Weight (N)." [1]

95 Plot the data points. [1]

96 Draw the best-fit line. [1]

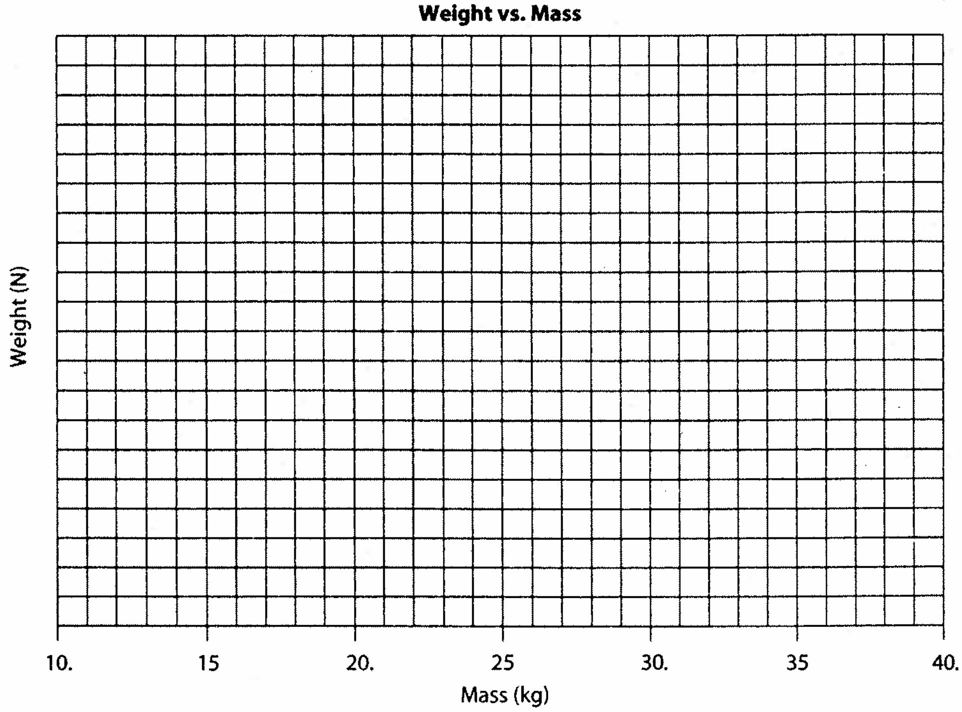
97 Using the graph, calculate the acceleration due to gravity on the planet. [1]

98 On the same grid, draw a line to represent acceleration due to gravity on Earth. [2]

Data Table

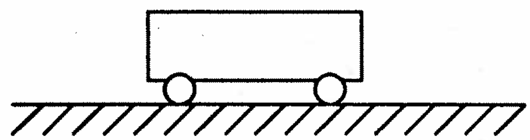
Mass (kilograms)	Weight (newtons)
15	106
20.	141
25	179
30.	216
35	249

Weight vs. Mass



Base your answers to questions 99 through 104 on the following information.

A child pulls a cart with rubber wheels at constant speed across a dry, horizontal concrete surface by exerting a force of 50. newtons at an angle of 35° above the horizontal.



99 On the diagram, use a protractor and a ruler to construct a vector to represent the 50.-newton force acting on the cart. Use a scale of 1.0 centimeter = 10. newtons. Label the vector 50.-newton force and the angle 35°. [2]

100 Construct the horizontal component of the force vector to scale on the diagram and label it *H*. [1]

101 Determine the magnitude of the horizontal component of the force. [1]

102 What is the magnitude of the frictional force between the cart's rubber wheels and the concrete? [1]

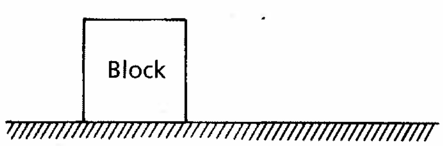
103 Determine the magnitude of the normal force on the cart. [2]

104 Compare the magnitude of the normal force acting on the cart to the weight of the cart. [1]

Base your answers to questions 105 through 107 on the following information.

A block weighing 4.2 newtons, sliding from left to right in a straight line on a horizontal surface, is acted upon by a 2.4-newton friction force. The block will be brought to rest by the friction force in a distance of 4.0 meters.

105 On the diagram below, draw an arrow to identify the direction of each force on the block while it is still moving but being slowed by the friction force. Identify each force by appropriately labeling the arrow that represents its line of direction. [3]

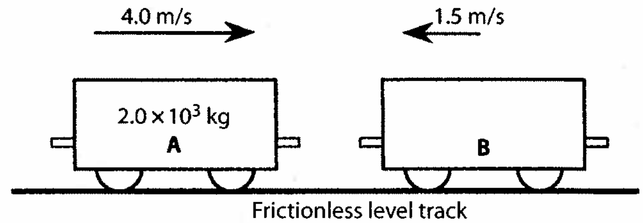


106 Calculate the magnitude of the acceleration of the block as it is brought to rest. [2]

107 Calculate the coefficient of friction between the two surfaces in contact. [2]

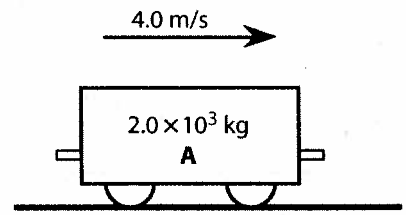
Base your answers to questions 108 through 110 on the following information and diagram.

Two railroad cars, A and B, are on a frictionless, level track. Car A has a mass of 2.0×10^3 kilograms and a velocity of 4.0 meters per second toward the right. Car B has a velocity of 1.5 meters per second toward the left. The magnitude of the momentum of cart B is 6.0×10^3 kilogram · meters per second. When the two cars collide, they lock together.



108 Calculate the magnitude of the momentum of car A before the collision. [2]

109 On the diagram below, construct a vector to represent the momentum of car A before the collision. Use a scale of 1.0 centimeter = 1.0×10^3 kilogram • meters per second [1]



110 Describe the momentum of the two cars after the collision and justify your answer based on the initial momentum of both cars. [2]

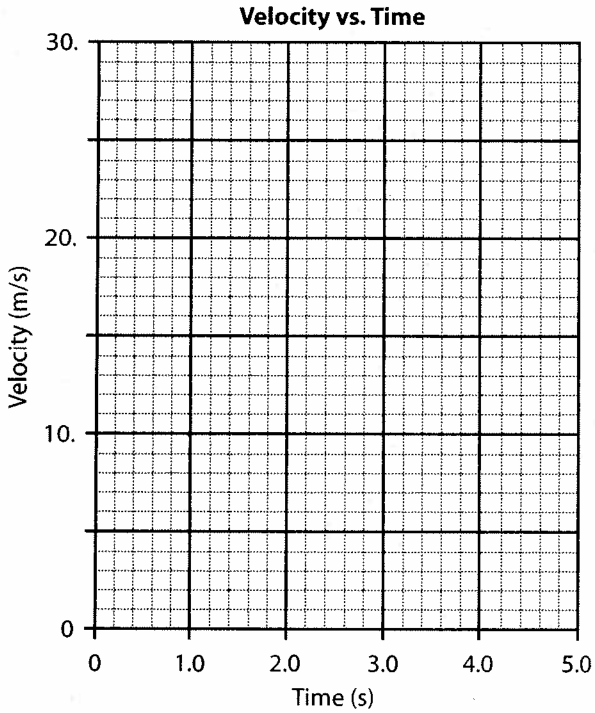
Base your answers to questions 111 through 117 on the following information and data table.

A 1500-kilogram car is traveling due north at 24.0 meters per second when the driver sees an obstruction on the highway. The data table shows the velocity of the car at 1.0-second intervals as it is brought to rest on the straight, level highway.

Data Table

Time (s)	Velocity (m/s)
0.0	24.0
1.0	19.0
2.0	14.0
3.0	10.0
4.0	4.0

111 Plot the data points for velocity versus time on the grid. [1]



112 Draw the best-fit line. [1]

113 Using your graph, calculate the acceleration of the car. [2]

114 Using your graph, calculate the total distance traveled by the car as it is brought to rest. [2]

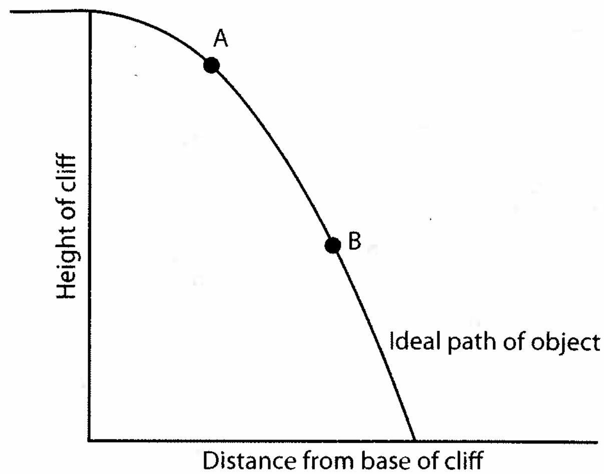
115 Calculate the magnitude of the car's total change in momentum as it is brought to rest. [2]

116 Calculate the magnitude and direction of the average force required to bring the car to rest. [2]

117 Compare the magnitude of the impulse imparted to the car to the magnitude of the car's change in momentum as it is brought to rest. [1]

Base your answers to questions 118 through 120 on the information and diagram below.

An object was projected horizontally from a tall cliff. The diagram below represents the path of the object, neglecting friction.



118 Compare the magnitude of the horizontal component of the object's velocity at point *A* to the magnitude of the horizontal component of the object's velocity at point *B*. [1]

119 Compare the magnitude of the vertical component of the object's velocity at point *A* to the magnitude of the vertical component of the object's velocity at point *B*. [1]

120 On the diagram sketch a likely path of the horizontally projected object, assuming it was subject to air resistance. [1]

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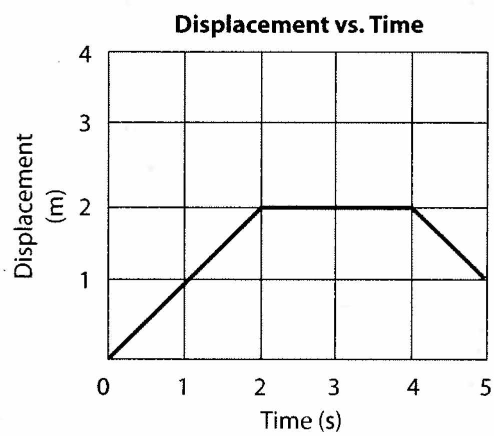
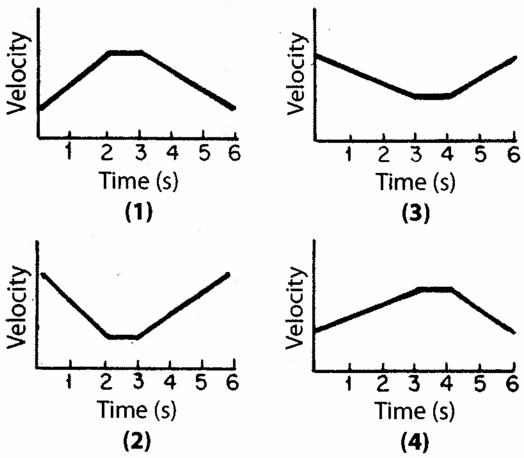
121 The graph below represents the motion of an object traveling in a straight line as a function of time.



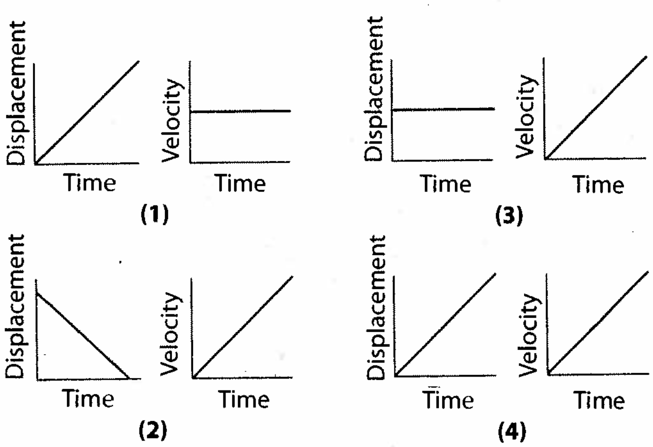
Determine the average speed of the object during the first 4.0 seconds. [1]

122 A group of bike riders took a 4.0-hour trip. During the first 3.0 hours they traveled a total of 50. kilometers, but during the last hour they traveled only 10. kilometers. Calculate the group's average speed for the entire trip. [2]

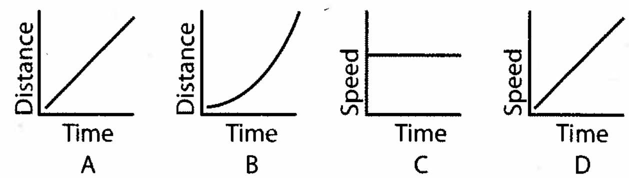
123 Which graph best represents the relationship between velocity and time for an object that has a uniform positive acceleration for 2 seconds, then moves at constant velocity for 1 second, and finally has a uniform negative acceleration for 3 seconds?



124 Which pair of graphs represents the same motion?

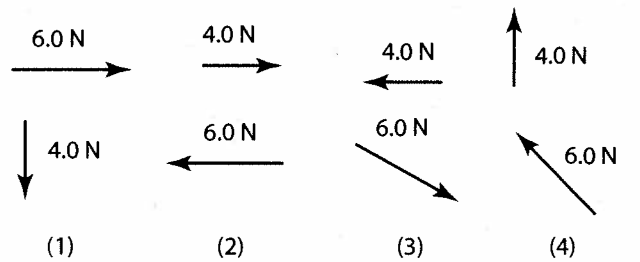


125 Which combination of graphs best describes the motion of a freely falling body? [Neglect friction.]

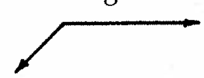


(1) A and C
(2) B and D
(3) A and D
(4) B and C

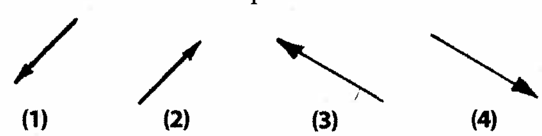
126 Which pair of forces acting concurrently on an object will produce the resultant of greatest magnitude?



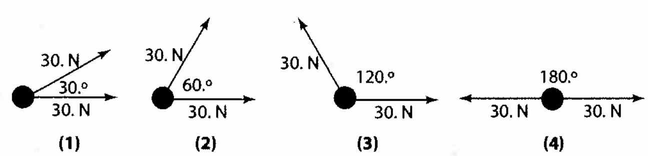
127 Two concurrent forces act on a point, as shown in the following vector diagram.



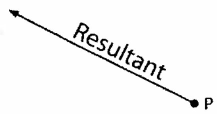
Which vector best represents their resultant?



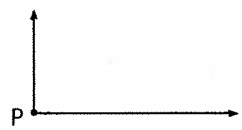
128 Two 30.-newton forces act concurrently on an object. In which diagram would the forces produce a resultant with a magnitude of 30. newtons?



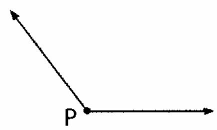
129 The vector that follows represents the resultant of two forces acting concurrently on an object at point P.



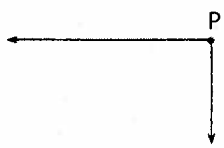
Which pair of vectors best represents two concurrent forces that combine to produce this resultant force vector?



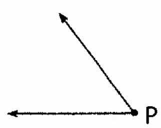
(1)



(3)

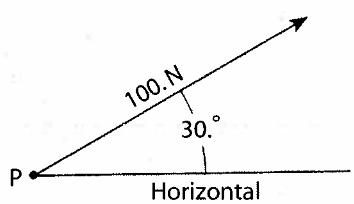


(2)



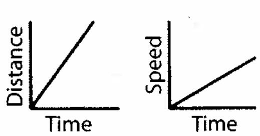
(4)

130 A 100.-newton force acts on point P as shown in the following diagram.

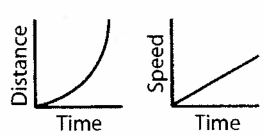


Determine the magnitude of the vertical component of this force. [1]

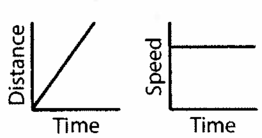
131 Which two graphs represent the motion of an object on which the net force is zero?



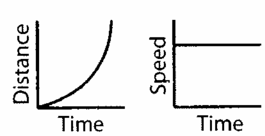
(1)



(3)

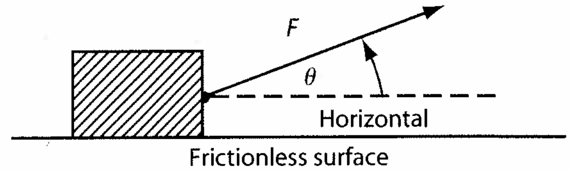


(2)



(4)

132 The following diagram represents a constant force acting on a box located on a frictionless horizontal surface.

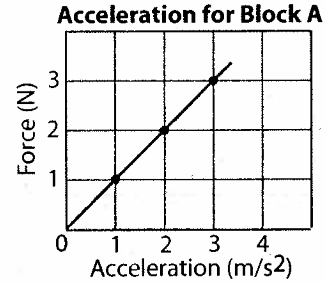


As the angle θ between the force and the horizontal is increased, the magnitude of the acceleration of the box

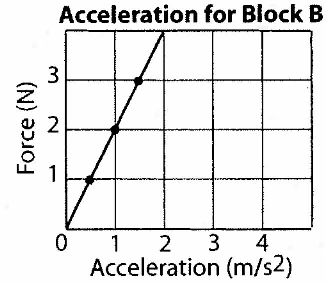
(1) decreases
(2) increases
(3) remains the same

133 A series of unbalanced forces was applied to each of two blocks A and B. The following graphs show the relationship between the magnitude of the unbalanced force and the magnitude of the acceleration for each block.

Unbalanced Force vs. Acceleration for Block A

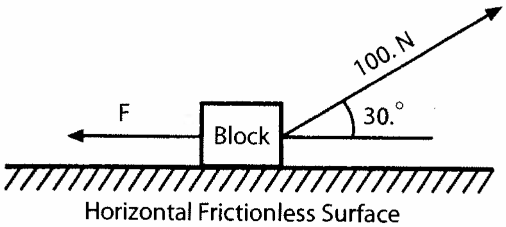


Unbalanced Force vs. Acceleration for Block B



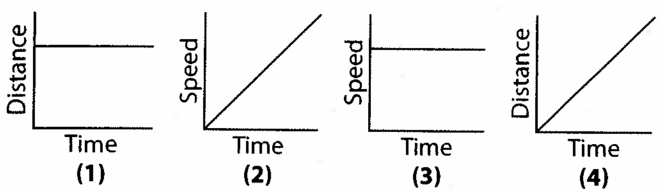
Compare the mass of block A to the mass of block B. [1]

134 The following diagram shows a block on a horizontal frictionless surface. A 100.-newton force acts on the block at an angle of 30.° above the horizontal.

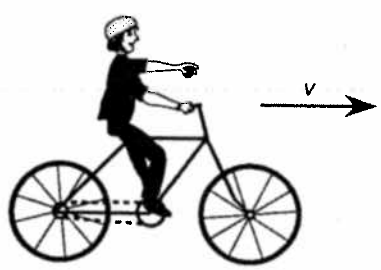


Calculate the magnitude of horizontal force *F* if it establishes equilibrium. [2]

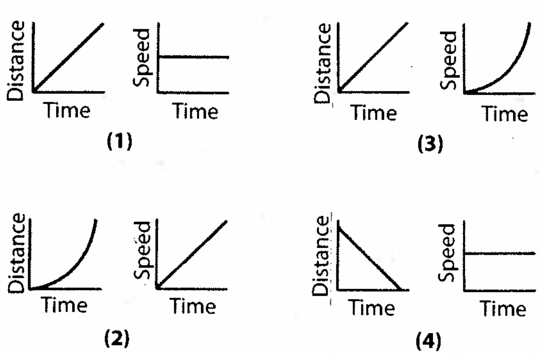
135 Which graph best represents the motion of an object that is *not* in equilibrium as it travels along a straight line?



136 In the following diagram, a cyclist traveling at constant speed to the right drops a ball from her hand.

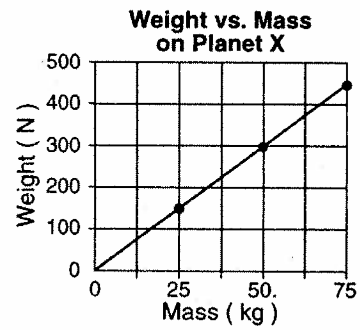


Which pair of graphs best represents the horizontal motion of the ball relative to the ground? [Neglect Friction.]



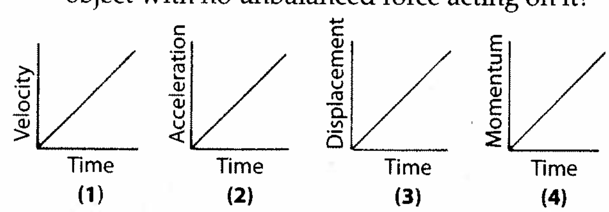
137 A projectile has an initial horizontal velocity of 15 meters per second and an initial vertical velocity of 25 meters per second. Determine the projectile's horizontal displacement if the total time of flight is 5.0 seconds. [Neglect friction.] [1]

138 The following graph shows the weight of three objects on planet X as a function of their masses.

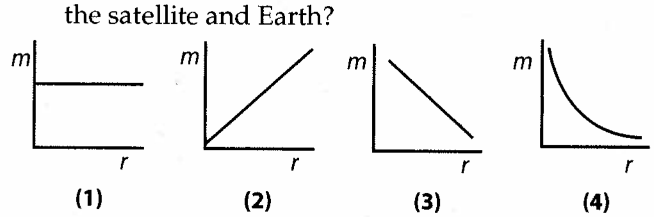


Determine the acceleration due to gravity on planet X. [1]

139 Which graph best represents the motion of an object with no unbalanced force acting on it?



140 Which graph best represents the relationship between the mass *m* of a satellite launched from Earth and the distance *r* between the centers of the satellite and Earth?



Part C

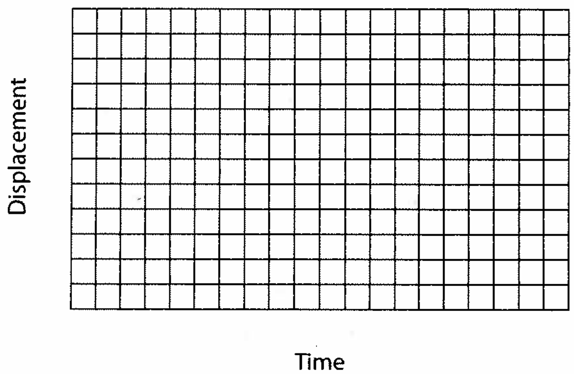
Base your answers to questions 141 and 142 on the following information.

A student walks from her house towards the bus stop, located 50. meters to the east. After walking 20. meters, she remembers that she left her lunch at the door. She runs home, picks up her lunch, walks again, and arrives at the bus stop.

141 On the grid, sketch a displacement versus time graph for the student's motion. [4]

142 Label the axes with appropriate values for time and displacement. [1]

Displacement vs. Time



Base your answers to questions 143 through 149 on the following information and data table.

Students decided to verify the value for acceleration due to gravity found in the *Reference Tables for Physical Setting/Physics* by performing a simple experiment. A ball bearing was dropped from the ceiling of the classroom and the time of fall measured. The twenty students took three measurements of the vertical distance and recorded an average value of 2.848 meters. Each student dropped the ball twice. The times of fall were recorded in the following tables, one showing the times as originally recorded and the other with the data sorted.

Unsorted

Time (s)	
0.97	0.87
0.86	0.75
1.00	0.68
0.81	0.72
0.98	0.78
0.77	0.77
0.87	0.80
0.87	0.75
0.88	0.77
0.71	0.78
0.73	0.69
0.72	0.69
0.78	0.68
0.76	0.87
0.75	0.75
0.76	0.50
0.69	0.57
0.83	0.67
0.56	0.66
0.80	0.61

Sorted

Time (s)	
0.50	0.76
0.56	0.77
0.57	0.77
0.61	0.77
0.66	0.78
0.67	0.78
0.68	0.78
0.68	0.80
0.69	0.80
0.69	0.81
0.69	0.83
0.71	0.86
0.72	0.87
0.72	0.87
0.73	0.87
0.75	0.87
0.75	0.88
0.75	0.97
0.75	0.98
0.76	1.00

143 What is the range of the data? [1]

144 Determine the mean of the data, to the nearest ten thousandth of a second and to the nearest hundredth of a second. [1]

145 Determine the standard deviation of the recorded values to the nearest hundredth of a second. [1]

146 What is the total number of values within one standard deviation from the mean? [1]

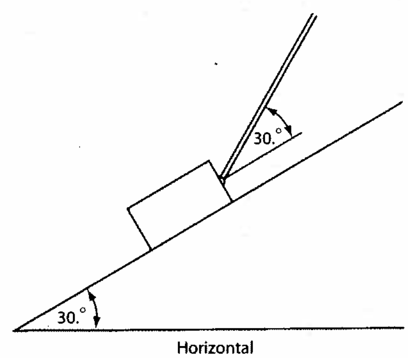
147 What percent of the data is within one standard deviation from the mean? [1]

148 Calculate the acceleration due to gravity on Earth based on the students' data. [2]

149 Determine the percent error. [1]

Base your answers to questions 150 through 153 on the following information and diagram.

A block weighing 100. newtons is positioned on an incline that makes an angle of 30.° with the horizontal. The magnitude of the friction force between the block and the incline is 10. newtons. A force of 120. newtons is applied by pulling on a rope that makes an angle of 30.° with the incline, as shown.



150 Draw a free-body diagram, and provide appropriate labels for each of the forces. [4]

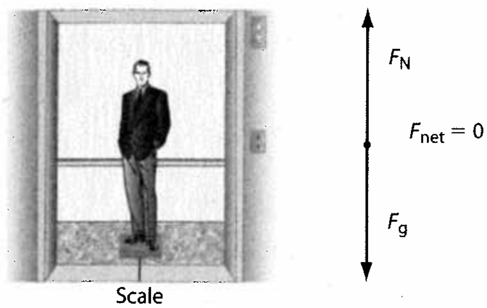
151 Calculate the component of the block's weight parallel to the incline. [2]

152 Calculate the magnitude and direction of the component of the tension that is useful in moving the block up the incline. [2]

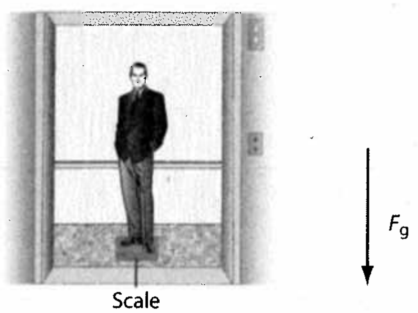
153 Calculate the magnitude and direction of the block's acceleration. [3]

Base your answers to questions 154 through 157 on the following information and diagrams.

A person standing on a scale in a stationary elevator weighs 735 newtons. The net force F_{net} on the person is zero because the normal force F_N is equal in magnitude but opposite in direction to the gravitational force F_g as shown.

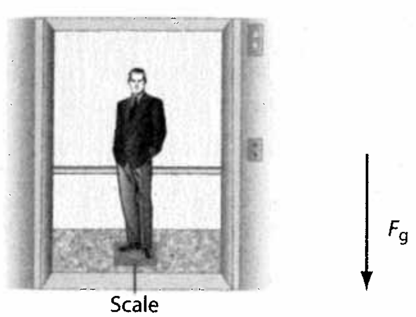


154 On the following diagram, sketch an arrow to indicate the relative magnitude of the normal force on the person if the elevator is moving downward at a constant speed of 2.5 meters per second. [1]



155 What is the net force on the person when the elevator is moving at constant speed downward? [1]

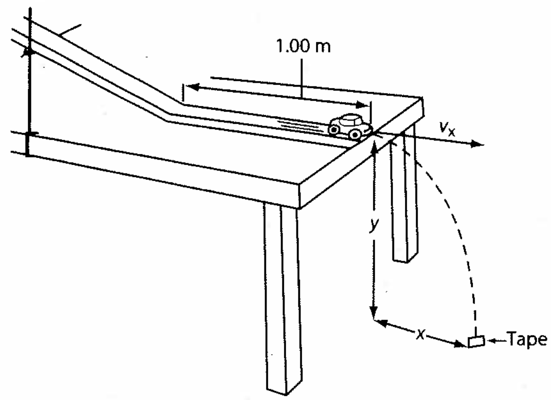
156 On the following diagram, sketch an arrow to indicate the relative magnitude of the normal force on the person if the elevator is accelerating upward at 2.5 meters per second^2. [1]



157 Compare the reading on the scale when the elevator is accelerating upward to the reading on the scale when the elevator is stationary. [1]

Base your answers to questions 158 through 162 on the following information and diagram.

Students performed an experiment to study horizontal projectile motion. A 2.00-meter-long flexible plastic track was positioned so that one end was at the edge of a lab table and the other end was elevated, as shown.



A toy car was started from rest from the elevated end of the track and the time for the car to travel the 1.00-meter horizontal distance to the edge of the table was measured. The times for three trials were 0.453 second, 0.347 second, and 0.390 second. The car was caught as it left the tabletop. Students measured the vertical distance from the bottom of the car on the horizontal track to the floor and recorded an average distance of 0.926 meter.

158 Calculate the average horizontal speed of the car. [2]

159 Calculate the time required for the car, initially at rest, to fall freely from the tabletop to the floor. [2]

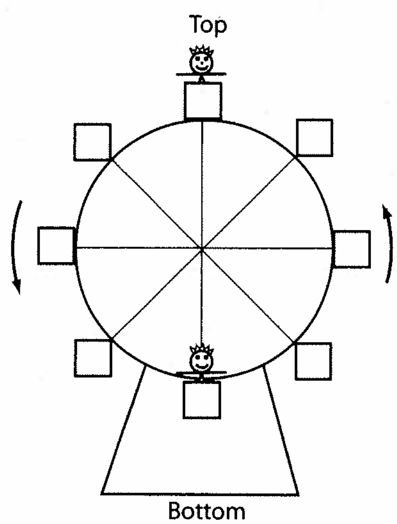
160 Calculate the horizontal distance the car would travel under these conditions if it wasn't caught. [2]

161 After having performed the calculations in questions 158 and 159, students measured the distance calculated in question 160 in a straight line from the horizontal track and directly beneath it on the floor. The spot was marked with a piece of masking tape. The car was then placed at the elevated end of the track and released from rest. State two reasons why the car landed about one centimeter short of the marked target. [2]

162 The experiment was repeated with the same car, but the elevated end of the track was positioned higher. Explain what effect releasing the car from rest at a greater height above the tabletop should have on (a) the average horizontal speed of the car [1], (b) the time required for the projected car to hit the floor after leaving the edge of the table [1], (c) the horizontal distance traveled by the car after it was projected from the tabletop [1].

Base your answers to questions 163 through 167 on the following information.

A child is moving at constant speed in a vertical circle on a ferris wheel. [Assume up is positive and down is negative.]



163 On the diagram, sketch and label arrows to represent the centripetal force, gravitational force, and normal force acting on the child at the bottom of the ride. [3]

164 On the diagram, sketch and label arrows to represent the three forces acting on the child at the top of the ride. [3]

165 Write an equation to show the relationship between the three forces acting on the child at the bottom of the ride. [1]

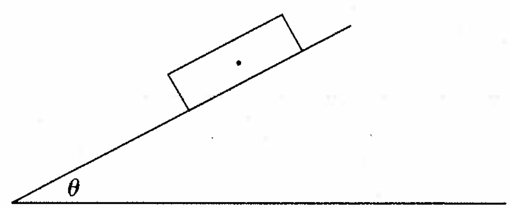
166 Write an equation to show the relationship between the three forces acting on the child at the top of the ride. [1]

---

167 The centripetal acceleration of a satellite in a circular orbit around Earth is produced by the gravitational force of attraction that the satellite exerts on Earth. Express the tangential speed v of the satellite in terms of the mass of Earth m_E and the distance r between the centers of the satellite and Earth. [2]

Base your answers to questions 168 through 170 on the information and diagram below.

A block of mass m slides at constant speed down a uniform plane inclined at an angle θ to the horizontal, as shown.



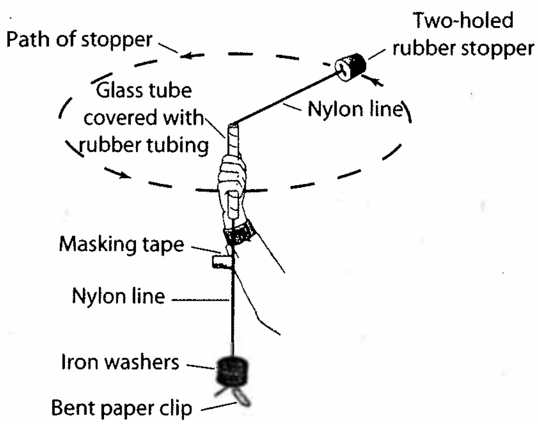
168 On the diagram, sketch and label arrows to represent the frictional force, the normal force, and the force of gravity acting on the block. [3]

169 Express the component of the block's weight parallel to the incline $F_{\parallel}$ and the component of the block's weight perpendicular to the incline $F_{\perp}$ in terms of F the weight of the block. [2]

170 Show that $\mu = \tan \theta$. [2]

Base your answers to questions 171 through 178 on the following information and diagram.

Two students, Julia and Tom, decided to perform an experiment to verify Newton's Second Law as applied to uniform circular motion; that is $F_c = \frac{mv^2}{r}$. In the lab, they collected the following materials: 15-centimeter long glass tube, fire-polished at each end and covered with rubber tubing, a piece of nylon line approximately one meter long, several two-holed rubber stoppers, a paper clip, masking tape, 36 identical iron washers, a stopwatch, and a triple-beam balance. An apparatus was assembled, with the intention of using it as shown below.



171 As the first objective Tom wrote: "To determine the relationship between the velocity of an object moving in a circular path and the magnitude of the centripetal force acting on the object." Julia objected stating they only could collect data that would enable them to determine the average speed of a rubber stopper in a circular path, but not its velocity. Identify which student was correct and explain why. [1]

172 Describe how the number of washers suspended at one end of the nylon cord could be converted into a measurement of the magnitude of the centripetal force acting on the stopper. [1]

173 To avoid having to use either the term velocity or speed, Tom changed their first objective to: "To determine the relationship between the period of revolution of an object moving in a circular path and the magnitude of the centripetal force acting on it." Using formulas found on the *Reference Tables for Physical Setting/Physics*, derive an expression for centripetal force in terms of r the radius of curvature of the path, m the object's mass, and T the period of revolution. [2]

174 List two *essential* pieces missing from their compilation of laboratory materials. [1]

175 The pair decided to time the motion of the stopper for thirty revolutions instead of making three separate trials of one revolution each and calculating an average to determine its period. Provide a rationale for this decision. [1]

176 *After securing **all** essential materials*, the pair proceeded with the collection of data. Julia practiced swinging the stopper overhead in a horizontal path, while keeping the radius of its path fixed. Once the technique was mastered the number of washers was varied for each trial from 36 to 4 washers in increments of 4 washers. Tom timed each event as noted in question 175. Identify four essential pieces of information that the students should have recorded in their data table. [2]

177 After the data was collected the students decided to graph their results. Sketch the general shape of the graph that should result for period of revolution versus magnitude of centripetal force. Label the axis with the dependent and independent variables. [2]

178 Explain why the students concluded at the end of the lab period that they had not had sufficient time to verify $F_c = \frac{mv^2}{r}$. [1]

Base your answers to questions 179 and 180 on the information below.

Friction provides the centripetal force that allows a 1,600-kilogram car to round a curve of radius 80. meters at a speed of 20. meters per second.

179 Calculate the minimum coefficient of friction needed between the tires and the road to round the curve. [4]

180 If the mass of the car were increased, how would that affect the maximum speed at which it could round the curve? [1]